Math 8 Unit 1 worksheet: Powers of powers, products and quotients (answer key)

ANSWERS AT A GLANCE

1. (a) w^{24} (b) $16x^{12}$ (c) $25a^4b^6$ (d) $\frac{x^4}{81}$ (e) $-64c^{15}$ (f) $0.001k^6$ (g) $-m^{10}n^5$ (h) $\frac{a^{15}}{32b^{20}}$

2. (a) 7 (b) 5 (c) −3 (d) 5 (e) $(a^3b^5)^4$

3. $100^3 = 10^6$

4. (a) 2^{30} bytes (b) about 10^9 bytes

5. (a) x^{12} (b) $9a^4$ (c) $8m^{15}$ (d) y^{10}

6. $4^7 = 2^{14}$, $8^5 = 2^{15}$, $16^3 = 2^{12}$, $2^{3^2} = 2^9$, $32^2 = 2^{10}$. From least to greatest: 2^{3^2}, 32^2, 16^3, 4^7, 8^5.

7. (a) $n = 7$ (b) $n = 6$ (c) $n = 4$

8. (a) $(2x)^3 = 1000$ and $2x^3 = 250$. (b) $(2x)^3 = 8x^3 = 4 \cdot 2x^3$, so it is always 4 times as large.

WORKED SOLUTIONS

1. Simplify. Any letter under a fraction bar is nonzero.

(a) $(w^3)^8$ (b) $(-2x^3)^4$ (c) $(5a^2b^3)^2$ (d) $\left(\frac{x}{3}\right)^4$

(e) $(-4c^5)^3$ (f) $(0.1k^2)^3$ (g) $(-m^2n)^5$ (h) $\left(\frac{a^3}{2b^4}\right)^5$

ANSWER

(a) w^{24} (b) $16x^{12}$ (c) $25a^4b^6$ (d) $\frac{x^4}{81}$ (e) $-64c^{15}$ (f) $0.001k^6$ (g) $-m^{10}n^5$ (h) $\frac{a^{15}}{32b^{20}}$

SOLUTION

(a) Eight copies of w^3 make $3 \cdot 8 = 24$ factors of w: $(w^3)^8 = w^{24}$.

(b) The fourth power goes to each factor: $(-2)^4(x^3)^4 = 16x^{12}$. Four negative factors pair off, so the result is positive.

(c) $5^2(a^2)^2(b^3)^2 = 25a^4b^6$.

(d) The fourth power goes to the top and to the bottom: $\frac{x^4}{3^4} = \frac{x^4}{81}$.

(e) $(-4)^3(c^5)^3 = -64c^{15}$. Three negative factors leave one unpaired, so the result is negative.

(f) $(0.1)^3(k^2)^3 = 0.001k^6$, since $0.1 \cdot 0.1 \cdot 0.1 = 0.001$.

(g) The product in parentheses is $(-1) \cdot m^2 \cdot n$, so $(-1)^5(m^2)^5n^5 = -m^{10}n^5$.

(h) Top: $(a^3)^5 = a^{15}$. Bottom: $(2b^4)^5 = 2^5(b^4)^5 = 32b^{20}$. So the answer is $\frac{a^{15}}{32b^{20}}$.

2. Fill in each blank so that the equation is true for every value of the letters. Any letter under a fraction bar is nonzero.

(a) $(x^{\square})^4 = x^{28}$

(b) $(2y^{\square})^3 = 8y^{15}$

(c) $(\square\, p^2)^3 = -27p^6$

(d) $\left(\frac{3}{k^{\square}}\right)^2 = \frac{9}{k^{10}}$

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(e) $(a^3b^{\square})^{\square} = a^{12}b^{20}$

ANSWER
(a) 7 (b) 5 (c) −3 (d) 5 (e) $(a^3b^5)^4$

SOLUTION
Work backward from the rule $(a^m)^n = a^{mn}$: the exponent inside times the exponent outside gives the exponent of the answer.

(a) The missing exponent times 4 is 28, so it is 7: $(x^7)^4 = x^{28}$.

(b) $2^3 = 8$ already, and the missing exponent times 3 is 15, so it is 5: $(2y^5)^3 = 8y^{15}$.

(c) The coefficient cubed must be −27. The only number whose cube is −27 is −3: $(-3p^2)^3 = -27p^6$.

(d) The bottom $k^?$ squared is k^{10}, so the missing exponent is 5: $\left(\frac{3}{k^5}\right)^2 = \frac{9}{k^{10}}$.

(e) The outside exponent times 3 is 12, so it is 4. Then the exponent on b times 4 is 20, so it is 5: $(a^3b^5)^4 = a^{12}b^{20}$.

3. Write $20^3 \cdot 5^3$ as a single power, and then as a power of 10.

ANSWER
$100^3 = 10^6$

SOLUTION
The two powers have different bases but the same exponent, so the power of a product, read from right to left, combines them:

$$20^3 \cdot 5^3 = (20 \cdot 5)^3 = 100^3$$

Since $100 = 10^2$, this is a power of a power, and the exponents multiply:

$$100^3 = (10^2)^3 = 10^{2 \cdot 3} = 10^6$$

4. In computing, a kilobyte is often counted as $2^{10} = 1024$ bytes. A megabyte is then 2^{10} kilobytes, and a gigabyte is 2^{10} megabytes.

(a) How many bytes are in a gigabyte? Give the answer as a single power of 2.

(b) Since $2^{10} = 1024$ is close to $1000 = 10^3$, estimate the number of bytes in a gigabyte as a power of 10.

ANSWER
(a) 2^{30} bytes (b) about 10^9 bytes

SOLUTION
(a) A gigabyte is 2^{10} megabytes, each of 2^{10} kilobytes, each of 2^{10} bytes. That is 2^{10} used as a factor three times:

$$2^{10} \cdot 2^{10} \cdot 2^{10} = (2^{10})^3 = 2^{10 \cdot 3} = 2^{30} \text{ bytes}$$

(b) Replace each 2^{10} by about 10^3:

$$(2^{10})^3 \approx (10^3)^3 = 10^9 \text{ bytes}$$

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That is about a billion bytes. (The exact value, $2^{30} = 1{,}073{,}741{,}824$, is a little more.)

5. Ethan simplified four expressions and got each one wrong. For each, say what he did wrong and write the correct answer.

(a) $(x^3)^4 = x^7$

(b) $(-3a^2)^2 = -9a^4$

(c) $(2m^5)^3 = 2m^{15}$

(d) $(y^2)^3 \cdot y^4 = y^{24}$

ANSWER

(a) x^{12} (b) $9a^4$ (c) $8m^{15}$ (d) y^{10}

SOLUTION

(a) He added the exponents. $(x^3)^4$ is four copies of x^3, one group repeated, so the exponents multiply: $x^{3 \cdot 4} = x^{12}$.

(b) He did not square the −3. The square goes to every factor in the parentheses, including the sign: $(-3)^2(a^2)^2 = 9a^4$.

(c) He did not cube the 2. The cube goes to every factor: $2^3(m^5)^3 = 8m^{15}$.

(d) He multiplied all three exponents. First $(y^2)^3 = y^6$, a power of a power. Then $y^6 \cdot y^4$ lines up two different groups, so those exponents add: $y^{6+4} = y^{10}$.

6. Write each number as a single power of 2. Then list the five numbers from least to greatest.

$$4^7, \quad 8^5, \quad 16^3, \quad 2^{3^2}, \quad 32^2$$

ANSWER

$4^7 = 2^{14}$, $8^5 = 2^{15}$, $16^3 = 2^{12}$, $2^{3^2} = 2^9$, $32^2 = 2^{10}$. From least to greatest: 2^{3^2}, 32^2, 16^3, 4^7, 8^5.

SOLUTION

Rewrite each base as a power of 2 and multiply the exponents:

- $4^7 = (2^2)^7 = 2^{14}$
- $8^5 = (2^3)^5 = 2^{15}$
- $16^3 = (2^4)^3 = 2^{12}$
- 2^{3^2} is worked from the top down: $3^2 = 9$, so it is 2^9.
- $32^2 = (2^5)^2 = 2^{10}$

With the same base 2, a larger exponent means more factors of 2, so a larger number. The exponents in order are 9, 10, 12, 14, 15, so the order is 2^{3^2}, 32^2, 16^3, 4^7, 8^5.

7. Find the whole number n that makes each equation true.

(a) $9^n = 3^{14}$

(b) $(4^n)^2 = 2^{24}$

(c) $8^n = 4^6$

ANSWER

(a) $n = 7$ (b) $n = 6$ (c) $n = 4$

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SOLUTION

Write both sides as powers of the same base. Two powers of 2 (or of 3) are equal only when their exponents are equal.

(a) $9^n = (3^2)^n = 3^{2n}$. So $2n = 14$ and $n = 7$.

(b) $(4^n)^2 = 4^{2n} = (2^2)^{2n} = 2^{4n}$. So $4n = 24$ and $n = 6$.

(c) $8^n = (2^3)^n = 2^{3n}$ and $4^6 = (2^2)^6 = 2^{12}$. So $3n = 12$ and $n = 4$.

8. (a) Find the value of $(2x)^3$ and of $2x^3$ when $x = 5$.

(b) Simplify $(2x)^3$. Then explain why, for every nonzero number x, the value of $(2x)^3$ is the same whole number of times the value of $2x^3$, and find that number.

ANSWER

(a) $(2x)^3 = 1000$ and $2x^3 = 250$. (b) $(2x)^3 = 8x^3 = 4 \cdot 2x^3$, so it is always 4 times as large.

SOLUTION

(a) $(2 \cdot 5)^3 = 10^3 = 1000$. In $2x^3$ only x is cubed: $2 \cdot 5^3 = 2 \cdot 125 = 250$.

(b) The cube goes to both factors in the parentheses: $(2x)^3 = 2^3x^3 = 8x^3$. In $2x^3$ the 2 is used once, but in $(2x)^3$ it is used three times. So

$$(2x)^3 = 8x^3 = 4 \cdot 2x^3$$

For any nonzero x, $(2x)^3$ is 4 times $2x^3$. Part (a) agrees: 1000 = 4 · 250.

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