Math 8 Unit 1 worksheet: Multiplying and dividing powers with the same base (answer key)

ANSWERS AT A GLANCE

1. (a) 3^7 (b) x^7 (c) d^9 (d) 12^1, which is 12 (e) u^{11} (f) v^8

2. (a) $n = 4$ (b) $n = 6$ (c) $n = 7$ (d) $n = 9$

3. (a) $2^6 = 64$ (b) $3^5 = 243$ (c) $7^1 = 7$ (d) $10^5 = 100{,}000$

4. (a) a^{3n} (b) c^{3n+1} (c) b^{n+4} (d) y^{2n}

5. (a) False: 5^9 (b) False: x^9 (c) True (d) False: the value is 18 (e) False: the value is 288 (f) True

6. (a) 3^8 pencils (b) 3^2 cartons

7. (a) Two equal terms make $2 \cdot 2^n = 2^1 \cdot 2^n = 2^{n+1}$. (b) $3^6 = 729$

WORKED SOLUTIONS

1. Simplify. Write each answer as a single power. The letters stand for nonzero numbers.

(a) $3^5 \cdot 3^2$

(b) $x^{11} \div x^4$

(c) $d \cdot d \cdot d^7$

(d) $\dfrac{12^{10}}{12^9}$

(e) $\dfrac{u^9}{u^2} \cdot u^4$

(f) $v^4 \cdot v^4$

ANSWER

(a) 3^7 (b) x^7 (c) d^9 (d) 12^1, which is 12 (e) u^{11} (f) v^8

SOLUTION

(a) Same base 3, so the exponents add: 5 + 2 = 7, giving 3^7.

(b) Same base x and 11 > 4, so the exponents subtract: 11 − 4 = 7, giving x^7.

(c) Each lone d is d^1, so the exponents add to 1 + 1 + 7 = 9: d^9.

(d) Same base 12 and 10 > 9: 10 − 9 = 1, so the quotient is 12^1, which is 12.

(e) First the quotient: 9 − 2 = 7, so $\dfrac{u^9}{u^2} = u^7$. Then the product: $u^7 \cdot u^4 = u^{11}$.

(f) The exponents add: 4 + 4 = 8, so v^8. Two rows of four factors make one row of eight, not sixteen.

2. Find the whole number n that makes each statement true. The letters p and x stand for nonzero numbers.

(a) $6^n \cdot 6^n \cdot 6^3 = 6^{11}$

(b) $\dfrac{p^{3n}}{p^n} = p^{12}$

(c) $\dfrac{x^{20}}{x^n \cdot x^4} = x^9$

(d) $2^n \cdot 16 = 2^{13}$

ANSWER

(a) $n = 4$ (b) $n = 6$ (c) $n = 7$ (d) $n = 9$

SOLUTION

(a) The exponents add: $n + n + 3 = 2n + 3$. Then $2n + 3 = 11$, so $2n = 8$ and $n = 4$. Check: $6^4 \cdot 6^4 \cdot 6^3 = 6^{4+4+3} = 6^{11}$.

(b) The exponents subtract: $3n - n = 2n$. Then $2n = 12$ and $n = 6$. Check: $\dfrac{p^{18}}{p^6} = p^{12}$.

(c) The bottom is x^{n+4}, so the left side is $x^{20-(n+4)}$. Then $20 - (n + 4) = 9$, so $n + 4 = 11$ and $n = 7$. Check: $\dfrac{x^{20}}{x^7 \cdot x^4} = \dfrac{x^{20}}{x^{11}} = x^9$.

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(d) $16 = 2^4$, so the left side is $2^n \cdot 2^4 = 2^{n+4}$. Then $n + 4 = 13$ and $n = 9$. Check: $2^9 \cdot 16 = 512 \cdot 16 = 8192 = 2^{13}$.

3. Write each as a single power of the base named, then find its value.

(a) $\dfrac{2^9 \cdot 4}{32}$, base 2

(b) $\dfrac{3^8}{27}$, base 3

(c) $\dfrac{49 \cdot 7^3}{7^4}$, base 7

(d) $1000 \cdot 10^4 \div 100$, base 10

ANSWER

(a) $2^6 = 64$ (b) $3^5 = 243$ (c) $7^1 = 7$ (d) $10^5 = 100{,}000$

SOLUTION

(a) $4 = 2^2$ and $32 = 2^5$, so the quotient is $\dfrac{2^9 \cdot 2^2}{2^5} = \dfrac{2^{11}}{2^5} = 2^6 = 64$. Check: 512 · 4 = 2048 and 2048 ÷ 32 = 64.

(b) $27 = 3^3$, so $\dfrac{3^8}{3^3} = 3^5 = 243$. Check: 6561 ÷ 27 = 243.

(c) $49 = 7^2$, so the top is $7^2 \cdot 7^3 = 7^5$, and $\dfrac{7^5}{7^4} = 7^1 = 7$.

(d) $1000 = 10^3$ and $100 = 10^2$, so the value is $10^{3+4} \div 10^2 = 10^7 \div 10^2 = 10^5 = 100{,}000$.

4. Here n is a positive whole number and the letters stand for nonzero numbers. Simplify each to a single power.

(a) $\dfrac{a^{5n}}{a^{2n}}$

(b) $c^n \cdot c^n \cdot c^{n+1}$

(c) $\dfrac{b^{2n+5}}{b^{n+1}}$

(d) $\dfrac{y^{4n} \cdot y^n}{y^{3n}}$

ANSWER

(a) a^{3n} (b) c^{3n+1} (c) b^{n+4} (d) y^{2n}

SOLUTION

Each exponent still counts factors, so the counts add and subtract as before.

(a) $5n > 2n$, and $5n - 2n = 3n$: a^{3n}.

(b) $n + n + (n + 1) = 3n + 1$: c^{3n+1}.

(c) $2n + 5 > n + 1$, and $(2n + 5) - (n + 1) = n + 4$: b^{n+4}.

(d) The top is $y^{4n+n} = y^{5n}$, and $5n - 3n = 2n$: y^{2n}. At $n = 1$ this says $\dfrac{y^4 \cdot y}{y^3} = y^2$, which agrees.

5. Decide whether each statement is true or false. Correct each false one: give the right single power, or the right value when no single power of that base fits. The letter x stands for a nonzero number.

(a) $5^3 \cdot 5^6 = 5^{18}$

(b) $\dfrac{x^{12}}{x^3} = x^4$

(c) $2^4 \cdot 2^4 = 2^8$

(d) $3^2 + 3^2 = 3^4$

(e) $2^5 \cdot 3^2 = 6^7$

(f) $\dfrac{10^8}{10^2} = 10^6$

ANSWER

(a) False: 5^9 (b) False: x^9 (c) True (d) False: the value is 18 (e) False: the value is 288 (f) True

SOLUTION

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(a) False. For a product the exponents add, 3 + 6 = 9, so it is 5^9. Multiplying them gave 18.

(b) False. For a quotient the exponents subtract, 12 − 3 = 9, so it is x^9. Dividing them gave 4.

(c) True. 4 + 4 = 8. As a check, $16 \cdot 16 = 256 = 2^8$.

(d) False. This is a sum, not a product, so nothing adds in the exponent. The value is 9 + 9 = 18, while $3^4 = 81$. Since 18 is $2 \cdot 3^2$, it is not a power of 3.

(e) False. The bases 2 and 3 differ, so the product rule does not apply. The value is 32 · 9 = 288, while $6^7 = 279{,}936$.

(f) True. Same base 10 and 8 − 2 = 6. As a check, 100,000,000 ÷ 100 = 1,000,000.

6. A warehouse holds 3^2 crates. Each crate holds 3^4 boxes, and each box holds 3^2 pencils.

(a) How many pencils are in the warehouse? Write the answer as a single power of 3.

(b) All the pencils are repacked into cartons that hold 3^6 pencils each. How many cartons are filled? Write the answer as a single power of 3.

ANSWER

(a) 3^8 pencils (b) 3^2 cartons

SOLUTION

(a) The number of pencils is crates times boxes per crate times pencils per box. All three are powers of 3, so the exponents add:

$$3^2 \cdot 3^4 \cdot 3^2 = 3^{2+4+2} = 3^8$$

(b) The number of cartons is the number of pencils divided by the pencils in one carton. The base is 3 in both and 8 > 6, so the exponents subtract:

$$\frac{3^8}{3^6} = 3^{8-6} = 3^2$$

7. (a) A sum of powers does not add its exponents. Even so, $2^n + 2^n = 2^{n+1}$ for every positive whole number n. Explain why.

(b) Write $3^5 + 3^5 + 3^5$ as a single power of 3, and find its value.

ANSWER

(a) Two equal terms make $2 \cdot 2^n = 2^1 \cdot 2^n = 2^{n+1}$. (b) $3^6 = 729$

SOLUTION

(a) Adding a number to itself doubles it, so $2^n + 2^n = 2 \cdot 2^n$. That is now a product of powers of 2, with $2 = 2^1$, so the product rule applies:

$$2 \cdot 2^n = 2^1 \cdot 2^n = 2^{n+1}$$

The exponent went up by 1 because one more factor of 2 joined the row. The exponents of the two terms in the sum were never added: $2^n \cdot 2^n$ would be 2^{2n}, which is different.

(b) Three equal terms make $3 \cdot 3^5 = 3^1 \cdot 3^5 = 3^6 = 729$. Check: 243 + 243 + 243 = 729.

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