Math 8 Unit 1 worksheet: Significant figures (answer key)

ANSWERS AT A GLANCE

1. (a) 4 (b) 3 (c) 3 (d) 3 (e) 2 (f) 3, 4 or 5 (g) 5 (h) 2
2. (a) 8.44 (b) 0.0060 (c) 4.0×10^5 (d) 20.0 (e) 0.8046 (f) 3.00×10^7
3. (a) 0.01 g (b) 0.0001 s (c) 100 m (d) 0.001 L, which is 1 mL (e) 0.01 km, which is 10 m
4. (a) 39.3 m^2 (b) 39.4 m^2; he rounded before the last step, and multiplying by 24 multiplied that rounding error.
5. (a) Owen: 1.15 m to 1.25 m. Layla: 1.195 m to 1.205 m. (b) No. The numbers are equal, but Layla's record claims a tool ten times finer.
6. (a) 3 and 3: the same. (b) 3, and 2 or 3: not clear; write 2.50×10^2 cm. (c) 2 and 2: the same. (d) 4 and 2: not the same; write 1.500×10^3 m.
7. 6.9 kg
8. 4.26 g
9. He kept every calculator digit; the area deserves three significant figures: 2.79×10^3 cm^2.

WORKED SOLUTIONS

1. Give the number of significant figures in each reading. Where the written number allows more than one count, list them all.

(a) 3.040 m (b) 0.0905 g (c) 7.10×10^{-3} kg (d) 508 mL

(e) 0.30 s (f) 40,500 km (g) 1.0060 L (h) 8.0×10^2 m

ANSWER

(a) 4 (b) 3 (c) 3 (d) 3 (e) 2 (f) 3, 4 or 5 (g) 5 (h) 2

SOLUTION

(a) Four. The 0 between 3 and 4 sits between nonzero digits, and the final 0 follows the decimal point.

(b) Three. The two zeros in front only place the point; the 0 between 9 and 5 counts.

(c) Three. In scientific notation every digit of 7.10 counts, including the final zero, and nothing else does.

(d) Three. The 0 sits between 5 and 8.

(e) Two. The leading 0 does not count; the trailing 0 after the point does.

(f) Three, four or five. The 4, 0 and 5 certainly count; the last two zeros are trailing zeros with no decimal point, so they may or may not have been measured.

(g) Five. Every zero here is either between nonzero digits or trailing after the point.

(h) Two: the digits of 8.0.

2. Round each number to the number of significant figures in parentheses. Use scientific notation when zeros at the end of a whole number would be ambiguous.

(a) 8.4362 (3) (b) 0.0059971 (2) (c) 395,210 (2)

(d) 19.96 (3) (e) 0.80455 (4) (f) 2.9998×10^7 (3)

ANSWER

(a) 8.44 (b) 0.0060 (c) 4.0×10^5 (d) 20.0 (e) 0.8046 (f) 3.00×10^7

SOLUTION

(a) The third significant figure is the hundredths digit 3; the next digit is 6, so round up: 8.44.

(b) The first significant figure is the 5, so two figures end at the ten-thousandths place. The next digit is 9, so 59 becomes 60: 0.0060. The final 0 must be written, or only one figure would show.

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(c) Two figures end at the ten-thousands place: 39 ten-thousands rounds up to 40, giving 400,000. Written plainly its zeros are ambiguous, so write 4.0×10^5.

(d) Three figures end at the tenths place. The next digit is 6, so 19.9 rounds up to 20.0; the zero in the tenths place is kept to show three figures.

(e) Four figures end at the ten-thousandths place, 0.8045, and the next digit is 5, which this course rounds up: 0.8046.

(f) Three figures of 2.9998 end at the hundredths place; the next digit is 9, so 2.99 rounds up to 3.00: 3.00×10^7, keeping both zeros.

3. Each reading below was written with exactly the digits its tool supported. Give the size of the tool's smallest step, with its unit.

(a) 6.30 g (b) 3.05×10^{-2} s (c) 4.0×10^3 m (d) 0.250 L (e) 7.200×10^1 km

ANSWER

(a) 0.01 g (b) 0.0001 s (c) 100 m (d) 0.001 L, which is 1 mL (e) 0.01 km, which is 10 m

SOLUTION

The last significant figure sits in the place of the tool's step.

(a) The last digit, 0, is in the hundredths place: steps of 0.01 g.

(b) $3.05 \times 10^{-2} = 0.0305$, whose last digit is in the ten-thousandths place: steps of 0.0001 s.

(c) $4.0 \times 10^3 = 4000$ with two significant figures, so the last one is the hundreds digit: steps of 100 m.

(d) The last digit is in the thousandths place: steps of 0.001 L, one milliliter.

(e) $7.200 \times 10^1 = 72.00$, so the last digit is in the hundredths place: steps of 0.01 km, ten meters.

4. A school roof holds 24 identical solar panels. One panel was measured as 1.65 m by 0.992 m.

(a) Find the total area of the 24 panels to an honest number of significant figures.

(b) Diego first rounds the area of one panel to three significant figures and then multiplies by 24. What does he report, and why does his answer differ from yours?

ANSWER

(a) 39.3 m^2 (b) 39.4 m^2; he rounded before the last step, and multiplying by 24 multiplied that rounding error.

SOLUTION

(a) One panel: $1.65 \times 0.992 = 1.6368$ m^2. All 24 panels: $24 \times 1.6368 = 39.2832$ m^2. Both measurements have three significant figures, and the 24 is a count, which is exact and does not limit. Keep three: 39.3 m^2.

(b) Diego rounds 1.6368 to 1.64, then 24 × 1.64 = 39.36, which he reports as 39.4 m^2. His early rounding added $1.64 - 1.6368 = 0.0032$ m^2 to every panel, and 24 panels turn that into about 0.077 m^2, enough to change the last digit. Carry every digit and round once, at the end.

5. Owen writes the height of a bookshelf as 1.2 m. Layla measures the same shelf and writes 1.20 m. Owen says the two records are the same, because 1.2 = 1.20.

(a) Give the smallest and largest true heights each record allows.

(b) Do the two records say the same thing? Explain.

ANSWER

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(a) Owen: 1.15 m to 1.25 m. Layla: 1.195 m to 1.205 m. (b) No. The numbers are equal, but Layla's record claims a tool ten times finer.

SOLUTION

(a) The last significant figure is good to within half a unit of its place. Owen's last figure is in the tenths place, so his record allows 1.2 ± 0.05: from 1.15 to 1.25 m. Layla's is in the hundredths place, so hers allows 1.20 ± 0.005: from 1.195 to 1.205 m.

(b) As numbers in arithmetic 1.2 and 1.20 are equal, but as records of a measurement they are not. Owen's two significant figures say his tool read tenths of a meter; Layla's three say hers read centimeters, so her range is ten times narrower and sits inside his.

6. Each pair records one measurement in two units. Give the number of significant figures in each record (every possibility if a number cannot say), and decide whether the two records claim the same precision. If they do not, rewrite the second record in scientific notation so that it matches the first.

(a) 0.0360 kg and 36.0 g

(b) 2.50 m and 250 cm

(c) 0.40 L and 4.0×10^2 mL

(d) 1.500 km and 1.5×10^3 m

ANSWER

(a) 3 and 3: the same. (b) 3, and 2 or 3: not clear; write 2.50×10^2 cm. (c) 2 and 2: the same. (d) 4 and 2: not the same; write 1.500×10^3 m.

SOLUTION

Changing the unit only moves the decimal point; it cannot add or remove a measured digit.

(a) 0.0360 has three (the leading zeros only place the point) and 36.0 has three. They match.

(b) 2.50 has three. 250 has a trailing zero with no decimal point, so it has two or three, and a reader cannot tell. Writing 2.50×10^2 cm shows all three.

(c) 0.40 has two and 4.0×10^2 has two. They match.

(d) 1.500 has four, but 1.5×10^3 has only two, so the meters record throws away two measured zeros. The match is 1.500×10^3 m.

7. Hannah packs a suitcase for a trip. The empty suitcase weighs 4.6 kg on a bathroom scale that reads tenths of a kilogram. A camera bag weighs 1.375 kg on a postal scale that reads to the gram, and a pair of boots weighs 0.88 kg on a scale that reads hundredths of a kilogram. Find the mass of the packed suitcase, reported honestly.

ANSWER

6.9 kg

SOLUTION

Add the three readings:

$$4.6 + 1.375 + 0.88 = 6.855$$

In a sum the coarsest last place decides. The suitcase reading reaches only tenths, so the total keeps tenths and no more: about 6.9 kg. If the tenths digit is a 0, it is still written, since the scale did show tenths.

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8. A metal tin holding 200 identical coins weighs 1.164 kg on a kitchen scale that reads to the gram. The empty tin weighs 0.312 kg on the same scale. Find the mass of one coin in grams, to an honest number of significant figures.

ANSWER

4.26 g

SOLUTION

The coins weigh 1.164 − 0.312 = 0.852 kg. Both readings reach thousandths, so the difference keeps thousandths: three significant figures, one fewer than either reading.

One coin: 0.852 ÷ 200 = 0.00426 kg. The 200 is a count, so it is exact and the quotient keeps the three figures of 0.852. In grams (1 kg = 1000 g, exact) that is 4.26 g.

9. Kevin measures a poster with a meter stick marked in millimeters: 61.0 cm by 45.7 cm. He reports its area as 2787.7 cm^2. Explain what is wrong with his report, and write the area so that a reader can tell exactly how many significant figures it has.

ANSWER

He kept every calculator digit; the area deserves three significant figures: 2.79×10^3 cm^2.

SOLUTION

61.0 × 45.7 = 2787.7 is the right arithmetic, but 2787.7 has five significant figures and claims the area to a tenth of a square centimeter. Each side has three, so the product keeps three, rounding at the tens place: 2790.

Written as 2790 cm^2, the final zero is ambiguous: a reader cannot tell whether it was measured. Scientific notation shows exactly three: 2.79×10^3 cm^2.

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