Math 8 Unit 1 worksheet: Zero and negative exponents (answer key)

ANSWERS AT A GLANCE

1. (a) 1 (b) $\frac{1}{81}$ (c) –27 (d) –16 (e) 4 (f) $\frac{1}{4}$ (g) 100 (h) 125

2. (a) $\frac{1}{m^7}$ (b) $\frac{2}{5k^3}$ (c) $\frac{1}{x^4y^4}$ (d) $\frac{b}{3a^2}$ (e) $\frac{7y^3}{2}$ (f) $\frac{z^2}{y^5}$ (g) $\frac{25}{p^2}$ (h) $\frac{6d^3}{c^4}$

3. (a) 1 (b) $\frac{1}{x^8}$ (c) y^5 (d) $\frac{1}{m^6}$ (e) $\frac{1}{t^4}$ (f) $\frac{p^6}{8}$ (g) $\frac{1}{k^3}$ (h) $\frac{b}{a}$

4. (a) $n=-5$ (b) $n=-3$ (c) $n=-4$ (d) $n=-6$ (e) $n=0$

5. $(-2)^{-3}=-\frac{1}{8}, 3^{-2}=\frac{1}{9}, 2^{-3}=\frac{1}{8}, 5^0=1, \left(\frac{1}{2}\right)^{-2}=4$

6. (a) 324 bacteria (b) $t=-5$, which is 5 hours before noon.

7. (a) $\frac{1}{9}$ (b) $\frac{5}{x^2}$ (c) 1 (d) 2

8. (a) True for $a=3$ ($\frac{1}{9}<9$); false for $a=\frac{1}{3}$ ($9>\frac{1}{9}$); false for $a=1$ (both are 1). (b) Exactly when $a>1$.

WORKED SOLUTIONS

1. Find the value of each. Give fractions in lowest terms.

(a) $\left(-\frac{3}{4}\right)^0$ (b) 3^{-4} (c) $\left(-\frac{1}{3}\right)^{-3}$ (d) $-\left(\frac{1}{2}\right)^{-4}$

(e) $\left(\frac{3}{2}\right)^{-2}\cdot 3^2$ (f) $(3^0+3)^{-1}$ (g) $\frac{1}{10^{-2}}$ (h) $(0.2)^{-3}$

ANSWER

(a) 1 (b) $\frac{1}{81}$ (c) –27 (d) –16 (e) 4 (f) $\frac{1}{4}$ (g) 100 (h) 125

SOLUTION

(a) The base $-\frac{3}{4}$ is nonzero, so its zero power is 1.

(b) $3^{-4}=\frac{1}{3^4}=\frac{1}{81}$.

(c) A fraction to a negative power turns over: $\left(-\frac{1}{3}\right)^{-3}=(-3)^3=-27$. Three negative factors make a negative product.

(d) The minus sign is outside the parentheses, so it is applied last: $-\left(\frac{1}{2}\right)^{-4}=-(2^4)=-16$.

(e) $\left(\frac{3}{2}\right)^{-2}=\left(\frac{2}{3}\right)^2=\frac{4}{9}$, and $\frac{4}{9}\cdot 9=4$.

(f) Inside the parentheses first: $3^0+3=1+3=4$. Then $4^{-1}=\frac{1}{4}$.

(g) The reciprocal of 10^{-2} is $10^2=100$.

(h) $0.2=\frac{1}{5}$, so $(0.2)^{-3}=\left(\frac{1}{5}\right)^{-3}=5^3=125$.

2. Rewrite with positive exponents only. Every variable is nonzero.

(a) m^{-7} (b) $\frac{2}{5}k^{-3}$ (c) $(xy)^{-4}$ (d) $\frac{a^{-2}}{3b^{-1}}$

(e) $\frac{7}{2y^{-3}}$ (f) $x^0y^{-5}z^2$ (g) $\left(\frac{p}{5}\right)^{-2}$ (h) $\frac{6c^{-4}d}{d^{-2}}$

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ANSWER

(a) $\dfrac{1}{m^7}$ (b) $\dfrac{2}{5k^3}$ (c) $\dfrac{1}{x^4y^4}$ (d) $\dfrac{b}{3a^2}$ (e) $\dfrac{7y^3}{2}$ (f) $\dfrac{z^2}{y^5}$ (g) $\dfrac{25}{p^2}$ (h) $\dfrac{6d^3}{c^4}$

SOLUTION

(a) By the definition, $m^{-7} = \dfrac{1}{m^7}$.

(b) The exponent belongs to k only, so only k^3 joins the 5 below: $\dfrac{2}{5} \cdot \dfrac{1}{k^3} = \dfrac{2}{5k^3}$.

(c) Here the base is the whole product xy: $(xy)^{-4} = \dfrac{1}{(xy)^4} = \dfrac{1}{x^4y^4}$.

(d) a^{-2} crosses to the bottom as a^2, and b^{-1} crosses to the top as b. The 3 has no negative exponent and stays where it is: $\dfrac{b}{3a^2}$.

(e) Only y carries the exponent, so $\dfrac{1}{y^{-3}} = y^3$ moves to the top and the 2 stays below: $\dfrac{7y^3}{2}$.

(f) $x^0 = 1$ and $y^{-5} = \dfrac{1}{y^5}$, so the product is $\dfrac{z^2}{y^5}$.

(g) Turn the fraction over and make the exponent positive: $\left(\dfrac{5}{p}\right)^2 = \dfrac{25}{p^2}$.

(h) Divide the d parts by subtracting exponents: $d^{1-(-2)} = d^3$. Then $c^{-4} = \dfrac{1}{c^4}$, so the answer is $\dfrac{6d^3}{c^4}$.

3. Simplify, writing the answer with positive exponents. Every variable is nonzero.

(a) $a^5 \cdot a^{-5}$ (b) $x^{-2} \cdot x^{-6}$ (c) $\dfrac{y^3}{y^{-2}}$ (d) $(m^{-3})^2$

(e) $\dfrac{t^{-7}}{t^{-3}}$ (f) $(2p^{-2})^{-3}$ (g) $(k^{-2})^0 \cdot k^{-3}$ (h) $\dfrac{(a^2b^{-1})^{-2}}{a^{-3}b}$

ANSWER

(a) 1 (b) $\dfrac{1}{x^8}$ (c) y^5 (d) $\dfrac{1}{m^6}$ (e) $\dfrac{1}{t^4}$ (f) $\dfrac{p^6}{8}$ (g) $\dfrac{1}{k^3}$ (h) $\dfrac{b}{a}$

SOLUTION

(a) Add the exponents: $a^{5+(-5)} = a^0 = 1$.

(b) $x^{-2+(-6)} = x^{-8} = \dfrac{1}{x^8}$.

(c) Subtract the exponents, minding the sign: $y^{3-(-2)} = y^5$.

(d) Multiply the exponents: $m^{(-3)(2)} = m^{-6} = \dfrac{1}{m^6}$.

(e) $t^{-7-(-3)} = t^{-4} = \dfrac{1}{t^4}$.

(f) The power goes to each factor: $2^{-3}(p^{-2})^{-3} = \dfrac{1}{8}p^6 = \dfrac{p^6}{8}$.

(g) Multiply the exponents in the first factor: $(k^{-2})^0 = k^0 = 1$. So the product is $k^{-3} = \dfrac{1}{k^3}$.

(h) Top: $(a^2b^{-1})^{-2} = a^{-4}b^2$. Then divide base by base: $a^{-4-(-3)}b^{2-1} = a^{-1}b = \dfrac{b}{a}$.

4. Find the integer n that makes each equation true. It may be positive, negative or zero.

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(a) $2^n = \dfrac{1}{32}$ (b) $10^n = 0.001$ (c) $\left(\dfrac{1}{3}\right)^n = 81$

(d) $5^n \cdot 5^3 = \dfrac{1}{125}$ (e) $(-7)^n = 1$

ANSWER

(a) $n = -5$ (b) $n = -3$ (c) $n = -4$ (d) $n = -6$ (e) $n = 0$

SOLUTION

Write both sides as powers of the same base and match the exponents.

(a) $\dfrac{1}{32} = \dfrac{1}{2^5} = 2^{-5}$, so $n = -5$.

(b) $0.001 = \dfrac{1}{1000} = \dfrac{1}{10^3} = 10^{-3}$, so $n = -3$.

(c) $81 = 3^4$, and $\left(\dfrac{1}{3}\right)^{-4} = 3^4$, so $n = -4$.

(d) The left side is 5^{n+3} and $\dfrac{1}{125} = 5^{-3}$, so $n + 3 = -3$ and $n = -6$.

(e) A nonzero number to the power zero is 1, so $n = 0$. No other integer works: a positive or negative power of −7 has size 7 or more, or $\dfrac{1}{7}$ or less.

5. Without a calculator, list these numbers from least to greatest. Show the value of each.

$$2^{-3}, \quad 3^{-2}, \quad (-2)^{-3}, \quad \left(\frac{1}{2}\right)^{-2}, \quad 5^0$$

ANSWER

$(-2)^{-3} = -\dfrac{1}{8}, 3^{-2} = \dfrac{1}{9}, 2^{-3} = \dfrac{1}{8}, 5^0 = 1, \left(\dfrac{1}{2}\right)^{-2} = 4$

SOLUTION

The values are $2^{-3} = \dfrac{1}{8}, 3^{-2} = \dfrac{1}{9}, (-2)^{-3} = \dfrac{1}{-8} = -\dfrac{1}{8}, \left(\dfrac{1}{2}\right)^{-2} = 2^2 = 4$ and $5^0 = 1$.

Only $(-2)^{-3}$ is negative (a negative base and an odd number of factors), so it is least. Of $\dfrac{1}{9}$ and $\dfrac{1}{8}$, ninths are smaller pieces, so $\dfrac{1}{9} < \dfrac{1}{8}$. Then come 1 and 4. The order is $(-2)^{-3}, 3^{-2}, 2^{-3}, 5^0, \left(\dfrac{1}{2}\right)^{-2}$.

6. At noon a lab culture holds 972 bacteria, and the number triples every hour. So t hours after noon the culture holds $972 \cdot 3^t$ bacteria, where a negative t stands for hours before noon.

(a) How many bacteria does the formula give at $t = -1$?

(b) For which t does the formula give 4 bacteria? How many hours before noon is that?

ANSWER

(a) 324 bacteria (b) $t = -5$, which is 5 hours before noon.

SOLUTION

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(a) By the definition of a negative exponent,

$$972 \cdot 3^{-1} = \frac{972}{3^1} = \frac{972}{3} = 324$$

Going back one hour divides by 3, just as going forward multiplies by 3.
(b) Write the noon count with a power of 3: $972 = 4 \cdot 3^5$. By the product rule,

$$972 \cdot 3^t = 4 \cdot 3^5 \cdot 3^t = 4 \cdot 3^{5+t}$$

This is 4 exactly when $3^{5+t} = 1 = 3^0$, so $5 + t = 0$ and $t = -5$: 5 hours before noon.

7. Each line has a mistake. Say what went wrong and give the correct value or expression. Every variable is nonzero.

(a) $3^{-2} = -9$

(b) $5x^{-2} = \dfrac{1}{5x^2}$

(c) $\dfrac{x^8}{x^8} = x^0 = 0$

(d) $\dfrac{1}{3^{-1} + 6^{-1}} = 3 + 6 = 9$

ANSWER

(a) $\dfrac{1}{9}$ (b) $\dfrac{5}{x^2}$ (c) 1 (d) 2

SOLUTION

(a) A negative exponent means a reciprocal, not a negative number: $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$.

(b) The exponent −2 belongs to x only, so only x moves below the bar: $5x^{-2} = \dfrac{5}{x^2}$. (The work shown is $(5x)^{-2}$.)

(c) The step x^0 is right, but $x^0 = 1$ for nonzero x, not 0. A nonzero number divided by itself is 1.

(d) A term of a sum may not cross the fraction bar. Work out the bottom first: $\dfrac{1}{3} + \dfrac{1}{6} = \dfrac{2}{6} + \dfrac{1}{6} = \dfrac{1}{2}$, so the value is $\dfrac{1}{\frac{1}{2}} = 2$.

8. A classmate says: "A negative exponent makes a number smaller, so a^{-2} is always less than a^2."

(a) Test the claim with $a = 3$, with $a = \dfrac{1}{3}$ and with $a = 1$.

(b) For which positive numbers a is the claim true? Explain.

ANSWER

(a) True for $a = 3$ ($\dfrac{1}{9} < 9$); false for $a = \dfrac{1}{3}$ ($9 > \dfrac{1}{9}$); false for $a = 1$ (both are 1). (b) Exactly when $a > 1$.

SOLUTION

(a) For $a = 3$: $3^{-2} = \dfrac{1}{9}$ and $3^2 = 9$, so $a^{-2} < a^2$. For $a = \dfrac{1}{3}$: $\left(\dfrac{1}{3}\right)^{-2} = 3^2 = 9$ and $\left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$, so here a^{-2} is the larger. For $a = 1$: both are 1, so neither is less.

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(b) $a^{-2} = \dfrac{1}{a^2}$, the reciprocal of a^2. A positive number is greater than its reciprocal exactly when it is greater than 1. If $a > 1$ then $a^2 > 1$, so $a^2 > \dfrac{1}{a^2}$ and the claim holds. If $a = 1$ the two are equal, and if $0 < a < 1$ then $a^2 < 1$ and its reciprocal is the larger. So the claim is true exactly when $a > 1$.

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