# Equations with the unknown on both sides

*In $5x + 3 = 2x + 18$ the unknown is on both sides, so undoing alone does not work. Collect the unknown on one side first, then finish as a two-step equation.*

## WORDS TO KNOW

**Coefficient:** the number that multiplies the letter. In $-7w$ it is $-7$.

**Equivalent equations:** equations with exactly the same solutions.

**Collect the unknown:** add or subtract a letter term on both sides so the letter is left on one side only.

**Substitution:** putting a number in place of the letter to see if both sides agree.

## KEY IDEAS

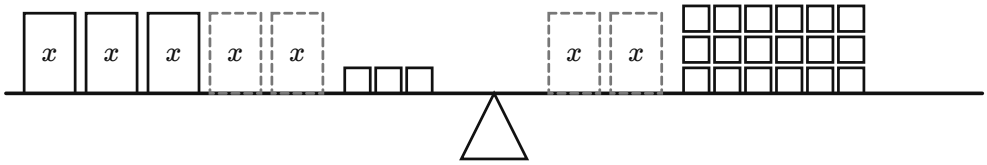
### 1. Take the same letter term off both sides

Whatever number $x$ is, $2x$ is a number too. Taking the same number off both sides keeps them equal, and adding it back undoes the step. So no solution is lost and none is added.

To solve $5x + 3 = 2x + 18$, subtract $2x$ from both sides:

$$3x + 3 = 18$$

Then it is a two-step equation: $3x = 15$, so $x = 5$.




$5x + 3$ $\qquad\qquad$ $2x + 18$

### 2. Move the term with the smaller coefficient

Either letter term can be moved, and both give the same answer. Moving the one with the **smaller** coefficient leaves a positive coefficient behind, so you never divide by a negative at the end. The letter may end up on the right: $5 = x$ is the same as $x = 5$.

"Smaller" means smaller as a signed number. In $4 - 3w = 9 - 7w$, $-7 < -3$, so you move $-7w$ by **adding** $7w$ to both sides.

### 3. The full method

1. Tidy each side on its own: distribute, combine like terms.
2. Collect the unknown on one side.
3. Collect the numbers on the other side.
4. Divide by the coefficient.
5. Check by substitution in the **original** equation: work out each side separately.

In a story, write each quantity with the same unknown. The moment the story asks about ("cost the same", "catches up") is when the two expressions are equal.


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## WATCH OUT

- **Smaller is about sign, not size.** $-8$ is smaller than $-3$, even though $8$ is bigger than $3$.
- **Clear parentheses before you move anything.** The left side of $3(x+1)=4x$ has no $3x$ term to move until you distribute.
- **A minus sign in front of parentheses hits every term.** $-(a-6)$ is $-a+6$.
- **Check in the original equation**, not a later line. A slip in an early step is copied into every line after it.
- **If the letter vanishes,** the two letter terms had the same coefficient. Then there is no solution or every number works, which is a different lesson.

## WORKED EXAMPLES

**Example 1. Solve $7-3w=-9w-53$.**

1. The coefficients are $-3$ and $-9$. Since $-9<-3$, move $-9w$ by adding $9w$ to both sides: $7+6w=-53$.
2. Subtract 7 from both sides: $6w=-60$.
3. Divide both sides by 6: $w=-10$.

**Answer:** $w=-10$. **Check:** the left side is $7-3(-10)=37$ and the right side is $-9(-10)-53=37$.

**Example 2. Solve $4(2a-3)-(a+5)=2(a+14)$.**

1. Tidy the left side. The first set of parentheses is multiplied by $4$, the second by $-1$:
$$8a-12-a-5=7a-17$$
2. Tidy the right side: $2(a+14)=2a+28$. The equation is now $7a-17=2a+28$.
3. Subtract $2a$ (the smaller term) from both sides: $5a-17=28$.
4. Add 17: $5a=45$. Divide by 5: $a=9$.

**Answer:** $a=9$. **Check:** left $4(15)-14=46$, right $2(23)=46$.

**Example 3. Gym A charges a 45 dollar joining fee plus 18 dollars a month. Gym B charges 9 dollars to join plus 22 dollars a month. After how many months have the two cost the same?**

1. Let $m$ be the number of months. The costs in dollars are $45+18m$ and $9+22m$.
2. They cost the same when $45+18m=9+22m$.
3. Subtract $18m$ (the smaller term): $45=9+4m$.
4. Subtract 9: $36=4m$. Divide by 4: $m=9$.

**Answer:** after 9 months, when each has cost 207 dollars. **Check:** $45+18(9)=207$ and $9+22(9)=207$.


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