Math 8 Unit 3 worksheet: Equations with variables on both sides (answer key)

ANSWERS AT A GLANCE

1. (a) $x = 6$ (b) $k = -5$ (c) $n = 4$ (d) $p = -6$ (e) $w = -5$ (f) $y = 5$ (g) $z = -\frac{8}{5}$ (h) $v = -4$
2. (a) $x = -9$ (b) $a = 8$ (c) $b = 4$ (d) $c = 12$ (e) $x = 10$ (f) $y = -12$
3. The third equation is wrong: subtracting 4 from both sides of $4x + 4 = -8$ gives $4x = -12$, not $4x = -4$. The solution is $x = -3$.
4. $2n + 30 = 5n - 6$, so the number is 12.
5. (a) $x = 5$; the square has side 11 cm and area 121 square cm. (b) $x = 16$; the length is 44 cm and the width 22 cm.
6. (a) $-5x$, because $-5 < -2$. (b) $x = 3$.
7. After 4 hours, when both candles are 4 cm tall.
8. $x = 3.5$; the triangle's sides are 10 cm, the square's sides are 7.5 cm, and each perimeter is 30 cm.

WORKED SOLUTIONS

1. Solve each equation. Move the variable term with the smaller coefficient, then finish as a two-step equation.

(a) $6x + 5 = 2x + 29$ (b) $3k - 7 = 8k + 18$ (c) $15 - 2n = 4n - 9$ (d) $9p = 4p - 30$

(e) $-5w + 2 = -8w - 13$ (f) $1.4y + 6 = 0.6y + 10$ (g) $2z - 5 = 7z + 3$ (h) $0.5v - 4 = 2v + 2$

ANSWER

(a) $x = 6$ (b) $k = -5$ (c) $n = 4$ (d) $p = -6$ (e) $w = -5$ (f) $y = 5$ (g) $z = -\frac{8}{5}$ (h) $v = -4$

SOLUTION

(a) Subtract $2x$: $4x + 5 = 29$, so $4x = 24$ and $x = 6$. Check: 41 = 41.

(b) Subtract $3k$: $-7 = 5k + 18$, so $-25 = 5k$ and $k = -5$. Check: −22 = −22.

(c) The coefficients are −2 and 4; add $2n$: $15 = 6n - 9$, so $24 = 6n$ and $n = 4$. Check: 7 = 7.

(d) Subtract $4p$: $5p = -30$, so $p = -6$. Check: −54 = −54.

(e) The coefficients are −5 and −8, and −8 is smaller; add $8w$: $3w + 2 = -13$, so $3w = -15$ and $w = -5$. Check: 27 = 27.

(f) Subtract $0.6y$: $0.8y + 6 = 10$, so $0.8y = 4$ and $y = 5$. Check: 13 = 13.

(g) Subtract $2z$: $-5 = 5z + 3$, so $-8 = 5z$ and $z = -\frac{8}{5}$. Check: both sides are $-\frac{41}{5}$.

(h) Subtract $0.5v$: $-4 = 1.5v + 2$, so $-6 = 1.5v$ and $v = -4$. Check: −6 = −6.

2. Solve each equation. Tidy each side first, then collect the unknown on one side.

(a) $3(x - 4) = 5x + 6$ (b) $2(a + 7) = 6(a - 3)$ (c) $4 - (b + 6) = 2(b - 7)$

(d) $6(c - 1) - 2c = 3(c + 2)$ (e) $\frac{5}{6}x - 3 = \frac{1}{3}x + 2$ (f) $\frac{1}{2}(8y - 6) = 5y + 9$

ANSWER

(a) $x = -9$ (b) $a = 8$ (c) $b = 4$ (d) $c = 12$ (e) $x = 10$ (f) $y = -12$

SOLUTION

(a) $3x - 12 = 5x + 6$. Subtract $3x$: $-12 = 2x + 6$, so $-18 = 2x$ and $x = -9$. Check: 3(−13) = −39 and 5(−9) + 6 = −39.

(b) $2a + 14 = 6a - 18$. Subtract $2a$: $14 = 4a - 18$, so $32 = 4a$ and $a = 8$. Check: 30 = 30.

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(c) The left side is $4 - b - 6 = -b - 2$, the right side $2b - 14$. Add b: $-2 = 3b - 14$, so $12 = 3b$ and $b = 4$. Check: 4 − 10 = −6 and 2(−3) = −6.

(d) The left side is $6c - 6 - 2c = 4c - 6$, the right side $3c + 6$. Subtract $3c$: $c - 6 = 6$, so $c = 12$. Check: 66 − 24 = 42 and 3(14) = 42.

(e) $\frac{1}{3} = \frac{2}{6}$ is smaller than $\frac{5}{6}$. Subtract $\frac{1}{3}x$: $\frac{1}{2}x - 3 = 2$, so $\frac{1}{2}x = 5$ and $x = 10$. Check: both sides are $\frac{16}{3}$.

(f) The left side is $4y - 3$. Subtract $4y$: $-3 = y + 9$, so $y = -12$. Check: $\frac{1}{2}(-102) = -51$ and 5(−12) + 9 = −51.

3. Kofi solved $7x + 4 = 3x - 8$ like this:

$$7x + 4 = 3x - 8 \quad \Rightarrow \quad 4x + 4 = -8 \quad \Rightarrow \quad 4x = -4 \quad \Rightarrow \quad x = -1$$

Find the step with the mistake and say what went wrong. Then give the correct solution and check it by substitution.

ANSWER

The third equation is wrong: subtracting 4 from both sides of $4x + 4 = -8$ gives $4x = -12$, not $4x = -4$. The solution is $x = -3$.

SOLUTION

The first step (subtract $3x$ from both sides) is right: $4x + 4 = -8$.

The second step should subtract 4 from both sides, and −8 − 4 = −12. Kofi wrote −4, as if he had added 4 on the right. So $4x = -12$, and dividing by 4 gives $x = -3$.

Check in the original: 7(−3) + 4 = −17 and 3(−3) − 8 = −17. They agree. (Kofi's $x = -1$ gives −3 on the left and −11 on the right, so a check would have caught it.)

4. Hannah thinks of a number. Multiplying it by 2 and then adding 30 gives the same result as multiplying it by 5 and then subtracting 6. Write an equation, and find the number.

ANSWER

$2n + 30 = 5n - 6$, so the number is 12.

SOLUTION

Let n be the number. Then $2n + 30 = 5n - 6$.

Subtract $2n$ (the smaller term) from both sides: $30 = 3n - 6$.

Add 6 to both sides: $36 = 3n$. Divide by 3: $n = 12$.

Check: 2 · 12 + 30 = 54 and 5 · 12 − 6 = 54.

5. A rectangle has length $(3x - 4)$ cm and width $(x + 6)$ cm.

(a) For what value of x is the rectangle a square? What is its area then?

(b) For what value of x is the length exactly twice the width? Give the length and width for that x.

ANSWER

(a) $x = 5$; the square has side 11 cm and area 121 square cm. (b) $x = 16$; the length is 44 cm and the width 22 cm.

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SOLUTION

(a) The rectangle is a square when its length equals its width:

$$3x - 4 = x + 6$$

Subtract x (the smaller term): $2x - 4 = 6$. Add 4: $2x = 10$, so $x = 5$. Then the length is 3(5) − 4 = 11 cm and the width is 5 + 6 = 11 cm, so it is a square of side 11 cm with area 11 · 11 = 121 square cm.

(b) The length is twice the width when

$$3x - 4 = 2(x + 6)$$

Distribute: $3x - 4 = 2x + 12$. Subtract $2x$: $x - 4 = 12$, so $x = 16$. Then the length is 3(16) − 4 = 44 cm and the width is 16 + 6 = 22 cm, and 44 = 2 · 22.

6. To solve $12 - 5x = 3 - 2x$, Sofia says the term to move is $-2x$, "because 2 is smaller than 5."

(a) Which x term has the smaller coefficient? Explain.

(b) Solve the equation by moving that term, and check your answer.

ANSWER

(a) $-5x$, because −5 < −2. (b) $x = 3$.

SOLUTION

(a) "Smaller" means smaller as a signed number. The coefficients are −5 and −2, and −5 < −2, so the term with the smaller coefficient is $-5x$. Sofia compared 5 and 2 without their signs.

(b) Moving $-5x$ means adding $5x$ to both sides: $12 = 3 + 3x$. Subtract 3: $9 = 3x$. Divide by 3: $x = 3$.

Check: 12 − 5(3) = −3 and 3 − 2(3) = −3. (Sofia's route also works, giving $12 - 3x = 3$, then $-3x = -9$ and $x = 3$, but with a negative coefficient to divide by.)

7. Two candles are lit at the same moment. The first is 16 cm tall and burns down 3 cm each hour. The second is 8 cm tall and burns down 1 cm each hour. After how many hours are the two candles the same height, and what is that height?

ANSWER

After 4 hours, when both candles are 4 cm tall.

SOLUTION

Let t be the number of hours since the candles were lit. The heights in cm are $16 - 3t$ and $8 - t$. They are the same height when

$$16 - 3t = 8 - t$$

The coefficients are −3 and −1, and −3 is the smaller. Add $3t$ to both sides: $16 = 8 + 2t$.

Subtract 8: $8 = 2t$. Divide by 2: $t = 4$.

Check: 16 − 3 · 4 = 4 and 8 − 1 · 4 = 4. Both candles are 4 cm tall after 4 hours, and neither has burned out, so the answer fits the story.

8. An equilateral triangle has sides of length $(2x + 3)$ cm. A square has sides of length $(x + 4)$ cm. The two shapes have the same perimeter. Find x, the side length of each shape, and the common perimeter.

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ANSWER

$x = 3.5$; the triangle's sides are 10 cm, the square's sides are 7.5 cm, and each perimeter is 30 cm.

SOLUTION

The triangle's perimeter is $3(2x + 3) = 6x + 9$ and the square's is $4(x + 4) = 4x + 16$. They are equal:

$$6x + 9 = 4x + 16$$

Subtract $4x$: $2x + 9 = 16$. Subtract 9: $2x = 7$. Divide by 2: $x = 3.5$.

The triangle's side is 2(3.5) + 3 = 10 cm and the square's side is 3.5 + 4 = 7.5 cm. Check: $3 \cdot 10 = 30$ and $4 \cdot 7.5 = 30$, so each perimeter is 30 cm.

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