Graph a line from its equation

You have the equation of a line, such as y = 3x - 5. How do you put its line on the grid without working out a whole table of values?

WORDS TO KNOW

Slope (m): the rise for each run of 1. As a fraction $\frac{p}{q}$, it means a run of q goes with a rise of p.

Run and rise: how far you move across (right is positive) and up (down is negative).

Horizontal line $y = c$: every point at height c. Its slope is 0.

y-intercept (b): where the line crosses the y-axis, the point $(0, b)$.

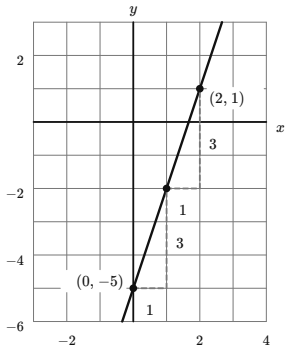
x-intercept: where the line crosses the x-axis, the point where $y = 0$.

Vertical line $x = c$: every point with x-coordinate c. It has no slope.

KEY IDEAS

1. Start at the intercept, then step by the slope

For $y = mx + b$, putting $x = 0$ leaves $y = b$, so $(0, b)$ is on the line. Along a line, the rise is always m times the run, wherever you start.



1. Plot the y-intercept $(0, b)$.
2. Write the slope as a fraction $\frac{p}{q}$ (so 3 is $\frac{3}{1}$). Step q right and p up, then step again.
3. Check one stepped point by substituting it into the equation.
4. Draw the line through the points, out to the edges of the grid.

The picture shows $y = 3x - 5$: start at (0, −5), then 1 right and 3 up, again and again.

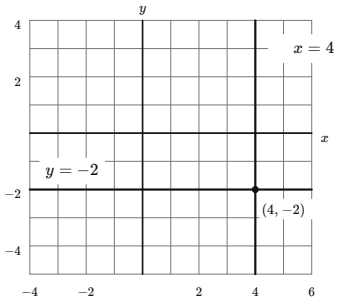
2. Negative slopes, fractions, and stepping backwards

A negative slope steps **down**: $m = -2$ means 1 right and 2 down. For a fraction, use the denominator as the run, so each step lands on a grid corner. A decimal slope becomes a fraction first: $0.4 = \frac{2}{5}$.

"Left q, down p" lands on the line just as "right q, up p" does. Step backwards when stepping forward would leave the grid.

3. Other forms, and lines with no x or no y

For $Ax + By = C$, there are two ways. Solve for y, dividing **every** term by the number in front of y. Or find the two intercepts: put $y = 0$ to find the x-intercept, and $x = 0$ to find the y-intercept.



$y = c$ is a horizontal line through $(0, c)$, with slope 0.

$x = c$ is a vertical line through $(c, 0)$. It has no slope and cannot be written as $y = mx + b$.

With big numbers, let each square stand for more than 1 unit, chosen for each axis on its own. The slope is still rise over run in **units**, not squares.

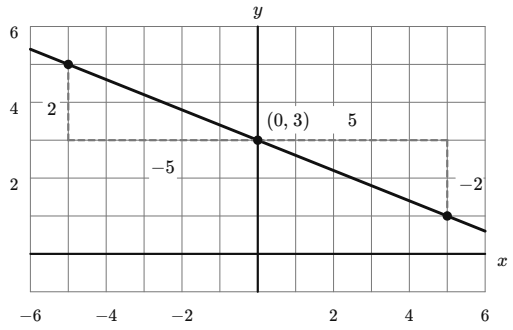
Free sample from math.wiki for classroom use. Do not post. math.wiki

WATCH OUT

- **The minus sign means down.** For a slope of $-\frac{p}{q}$, step q right and p down (or q left and p up), never q right and p up.
- **Check in the original equation.** A wrong rewrite still gives points that fit the wrong line. Substituting into the first equation tests the rewrite too.
- **Two points cannot warn you of a slip.** Any two points have a line through them. The third point is your check.
- **The intercept method needs two different points.** If $C = 0$, both intercepts are the origin, so find a second point another way.

WORKED EXAMPLES

Example 1. Graph $y = -\frac{2}{5}x + 3$.



1. Here $m = -\frac{2}{5}$ and $b = 3$. Plot (0, 3).
2. Read the slope as $\frac{-2}{5}$: 5 right and 2 down lands on (5, 1).
3. Read it as $\frac{2}{-5}$: 5 left and 2 up lands on (−5, 5).
4. Check: $-\frac{2}{5}(5) + 3 = -2 + 3 = 1$ and $-\frac{2}{5}(-5) + 3 = 2 + 3 = 5$.

Answer: the line through (−5, 5), (0, 3) and (5, 1). It falls gently, 2 down for every 5 across.

Example 2. Graph $5x + 2y = 6$.

1. Solve for y. Subtract $5x$: $2y = -5x + 6$. Divide every term by 2:
$$y = -\frac{5}{2}x + 3$$
2. Plot (0, 3). Step 2 right and 5 down to (2, −2). Step 2 left and 5 up to (−2, 8).
3. Check (2, −2) in the original equation: 5(2) + 2(−2) = 10 − 4 = 6.

Answer: the line through (−2, 8), (0, 3) and (2, −2).

Example 3. Graph $3x + 5y = 15$ from its two intercepts.

1. On the x-axis, $y = 0$: $3x = 15$, so $x = 5$. The point is (5, 0).
2. On the y-axis, $x = 0$: $5y = 15$, so $y = 3$. The point is (0, 3).
3. Third point: $x = -5$ gives $-15 + 5y = 15$, so $5y = 30$ and $y = 6$. The point (−5, 6) lies on the same line.

Answer: the line through (−5, 6), (0, 3) and (5, 0). Its slope is $-\frac{3}{5}$, a rise of −3 over a run of 5.

Free sample from math.wiki for classroom use. Do not post. math.wiki