Math 8 Unit 5 worksheet: Graphing a line from its equation (answer key)

ANSWERS AT A GLANCE

1. (a) (4, 13) (b) (9, 5) (c) (−4, 14) (d) (10, −4) (e) (−12, −21)
2. (a) $y = -1$, $y = 4$, $x = -3$, $x = 5$. (b) $y = -1$ and $y = 4$ have slope 0; $x = -3$ and $x = 5$ have no slope. (c) Only $y = -1$ and $y = 4$.
3. (a) At (0, 6); it falls. (b) Two steps; it crosses the x-axis at (8, 0). (c) Quadrant III.
4. (a) (−9, −10) (b) $y = \frac{2}{3}x - 4$ (c) Lucas draws Mei's line; Kofi does not.
5. (a) 2(9 − 1) = 16 but 3(1) + 8 = 11, so (1, 9) is not on the line: the rewrite is wrong. (b) He did not divide $5x$ by 2. The line is $y = \frac{5}{2}x + 4$, with y-intercept (0, 4) and stepped points (2, 9) and (−2, −1).
6. (a) The y-intercept $\left(0, \frac{2}{5}\right)$ is not a grid corner, so steps from it land between grid lines. (b) (1, 1) (c) (6, 4) and (−4, −2).
7. (a) At (0, −72) and at (12, 0): the submarine reaches the surface after 12 minutes. (b) After 7 minutes. (c) 4 squares right and 2 squares up.

WORKED SOLUTIONS

1. Each line is drawn by starting at its y-intercept and stepping by the slope, with the denominator of the slope as the run. Find the point you land on after the steps given, then check it by substitution.

(a) $y = 3x + 1$; 4 steps to the right

(b) $y = \frac{2}{3}x - 1$; 3 steps to the right

(c) $y = -\frac{5}{2}x + 4$; 2 steps to the left

(d) $y = -0.6x + 2$; 2 steps to the right

(e) $y = \frac{7}{4}x$; 3 steps to the left

ANSWER

(a) (4, 13) (b) (9, 5) (c) (−4, 14) (d) (10, −4) (e) (−12, −21)

SOLUTION

(a) Start at (0, 1). The slope $3 = \frac{3}{1}$: each step is 1 right and 3 up. Four steps make 4 right and 12 up, landing on (4, 13). Check: 3(4) + 1 = 13.

(b) Start at (0, −1). Each step is 3 right and 2 up, so three steps make 9 right and 6 up: (9, 5). Check: $\frac{2}{3}(9) - 1 = 6 - 1 = 5$.

(c) Start at (0, 4). Read $-\frac{5}{2}$ as $\frac{5}{-2}$: each step is 2 left and 5 up. Two steps make 4 left and 10 up: (−4, 14). Check: $-\frac{5}{2}(-4) + 4 = 10 + 4 = 14$.

(d) First write the slope as a fraction: $-0.6 = -\frac{3}{5}$. Start at (0, 2); each step is 5 right and 3 down. Two steps make 10 right and 6 down: (10, −4). Check: −0.6(10) + 2 = −6 + 2 = −4.

(e) Here $b = 0$, so start at the origin (0, 0). Stepping left, each step is 4 left and 7 down. Three steps make 12 left and 21 down: (−12, −21). Check: $\frac{7}{4}(-12) = -21$.

2. A rectangle has corners (−3, −1), (5, −1), (5, 4) and (−3, 4).

(a) Write an equation for the line through each of its four sides.

(b) Give the slope of each of the four lines, or say that it has none.

(c) Which of the four lines can be written in the form $y = mx + b$?

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ANSWER

(a) $y = -1$, $y = 4$, $x = -3$, $x = 5$. (b) $y = -1$ and $y = 4$ have slope 0; $x = -3$ and $x = 5$ have no slope. (c) Only $y = -1$ and $y = 4$.

SOLUTION

(a) The bottom side joins (−3, −1) and (5, −1). Both points have height −1, so the side lies on the horizontal line $y = -1$. In the same way the top side lies on $y = 4$. The left side joins (−3, −1) and (−3, 4), which share $x = -3$, so it lies on the vertical line $x = -3$; the right side lies on $x = 5$.

(b) Along a horizontal line the rise is 0 for every run, so $y = -1$ and $y = 4$ have slope 0. Along a vertical line the run is 0, and dividing by 0 gives no number, so $x = -3$ and $x = 5$ have no slope.

(c) The horizontal lines fit the form with $m = 0$: $y = 0x - 1$ and $y = 0x + 4$. The vertical lines have no slope to put in for m, so they cannot be written as $y = mx + b$.

3. Answer these questions about the line $y = -\frac{3}{4}x + 6$ from its equation, by stepping on a grid.

(a) Where does the line cross the y-axis, and does it rise or fall from left to right?

(b) Step by the slope from the y-intercept until you reach the x-axis. How many steps does that take, and where does the line cross the x-axis?

(c) The axes split the grid into four quadrants. Which quadrant does the line never pass through? Explain using your answers to (a) and (b).

ANSWER

(a) At (0, 6); it falls. (b) Two steps; it crosses the x-axis at (8, 0). (c) Quadrant III.

SOLUTION

(a) Here $b = 6$, so the line crosses the y-axis at (0, 6). The slope $-\frac{3}{4}$ is negative, so the line falls from left to right.

(b) Each step is 4 right and 3 down. From (0, 6): one step reaches (4, 3), a second reaches (8, 0), which is on the x-axis. Check: $-\frac{3}{4}(8) + 6 = -6 + 6 = 0$.

(c) The line crosses the positive y-axis at (0, 6) and the positive x-axis at (8, 0). Between those points it is in quadrant I. To the left of (0, 6) it climbs higher, so it is in quadrant II; to the right of (8, 0) it falls below the x-axis into quadrant IV. A point in quadrant III has $x < 0$, but every point of the line with $x < 0$ is above $y = 6$. So the line never enters quadrant III.

4. Mei draws a line by starting at (0, −4) and repeating the step "3 left and 2 down".

(a) Which point does she reach after three steps?

(b) Write the equation of her line in slope-intercept form, and check your point from (a) in it.

(c) Lucas also starts at (0, −4) but repeats the step "3 right and 2 up". Kofi starts at (0, −4) and repeats "2 right and 3 up". Which of them draws Mei's line? Explain.

ANSWER

(a) (−9, −10) (b) $y = \frac{2}{3}x - 4$ (c) Lucas draws Mei's line; Kofi does not.

SOLUTION

(a) Each step is 3 left and 2 down. From (0, −4): (−3, −6), then (−6, −8), then (−9, −10).

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(b) The line crosses the y-axis at −4, so $b = -4$. A run of −3 goes with a rise of −2, so the slope is $\frac{-2}{-3} = \frac{2}{3}$. The line is $y = \frac{2}{3}x - 4$. Check: $\frac{2}{3}(-9) - 4 = -6 - 4 = -10$.

(c) Lucas's step has rise 2 over run 3, the same slope $\frac{2}{3}$, from the same starting point, so he draws the same line (stepping right instead of left). His first point (3, −2) checks: $\frac{2}{3}(3) - 4 = -2$. Kofi's step has rise 3 over run 2: he swapped the rise and the run, so his slope is $\frac{3}{2}$. His first point (2, −1) is not on Mei's line, since $\frac{2}{3}(2) - 4 = -\frac{8}{3}$, not −1. His line meets Mei's only at (0, −4).

5. Noah wants to graph $2(y - x) = 3x + 8$. He rewrites it as $y = 5x + 4$, plots (0, 4), steps 1 right and 5 up to (1, 9), and checks: 5(1) + 4 = 9. So he is sure his graph is right.

(a) Check the point (1, 9) in the original equation instead. What does this show?

(b) Find Noah's mistake. Rewrite the equation correctly, then give the y-intercept and two points found by stepping. Check one of them in the original equation.

ANSWER

(a) 2(9 − 1) = 16 but 3(1) + 8 = 11, so (1, 9) is not on the line: the rewrite is wrong. (b) He did not divide $5x$ by 2. The line is $y = \frac{5}{2}x + 4$, with y-intercept (0, 4) and stepped points (2, 9) and (−2, −1).

SOLUTION

(a) In the original equation, the left side is 2(9 − 1) = 16 and the right side is 3(1) + 8 = 11. They differ, so (1, 9) is not on the line. Noah's check passed only because he checked in his rewritten equation, which tests his stepping but not his rewriting.

(b) Distribute and collect the x terms, then divide **every** term by 2:

$$\begin{aligned} 2y - 2x &= 3x + 8 \\ 2y &= 5x + 8 \\ y &= \tfrac{5}{2}x + 4 \end{aligned}$$

Noah divided the 8 by 2 but not the $5x$. The y-intercept is (0, 4). Step 2 right and 5 up to (2, 9), and 2 left and 5 down to (−2, −1).

Check (2, 9) in the original: 2(9 − 2) = 14 and 3(2) + 8 = 14. It agrees. (So does (−2, −1): 2(−1 + 2) = 2 and −6 + 8 = 2.)

6. Look at the line $y = \frac{3}{5}x + \frac{2}{5}$.

(a) Explain why the y-intercept is a poor point to start stepping from.

(b) Find the one point on the line whose coordinates are whole numbers and whose x-coordinate is from 1 to 5.

(c) From that point, step by the slope once to the right and once to the left. Check one of the stepped points by substitution.

ANSWER

(a) The y-intercept $\left(0, \frac{2}{5}\right)$ is not a grid corner, so steps from it land between grid lines. (b) (1, 1) (c) (6, 4) and (−4, −2).

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SOLUTION

(a) The y-intercept is $\left(0, \frac{2}{5}\right)$, two fifths of a square above the origin. It can only be placed by estimating, and every point stepped from it is off the grid corners by the same amount.

(b) Try each x: at $x = 1$, $y = \frac{3}{5} + \frac{2}{5} = 1$. At $x = 2, 3, 4, 5$ the outputs are $\frac{8}{5}$, $\frac{11}{5}$, $\frac{14}{5}$ and $\frac{17}{5}$, none of them whole. So the point is (1, 1).

(c) Each step is 5 right and 3 up, or 5 left and 3 down. From (1, 1) that gives (6, 4) and (−4, −2). Check (6, 4): $\frac{3}{5}(6) + \frac{2}{5} = \frac{18}{5} + \frac{2}{5} = 4$. (And (−4, −2): $-\frac{12}{5} + \frac{2}{5} = -2$.)

7. A research submarine is 72 meters below the surface of the sea and rises at a steady 6 meters per minute. Its height above the surface after t minutes is $h = 6t - 72$ meters, where a negative height means below the surface.

(a) Where does the graph cross the h-axis, and where does it cross the t-axis? Say what the second crossing means.

(b) After how many minutes is the submarine 30 meters below the surface?

(c) Hannah draws the graph with each square across standing for 3 minutes and each square up standing for 36 meters. Going along the line from the h-intercept to the t-axis, how many squares right and how many squares up does it move?

ANSWER

(a) At (0, −72) and at (12, 0): the submarine reaches the surface after 12 minutes. (b) After 7 minutes. (c) 4 squares right and 2 squares up.

SOLUTION

(a) Put $t = 0$: $h = -72$, the point (0, −72), where the submarine starts. Put $h = 0$: $6t = 72$, so $t = 12$, the point (12, 0). Height 0 is the surface, so the submarine reaches the surface after 12 minutes.

(b) 30 meters below the surface is $h = -30$: $6t - 72 = -30$, so $6t = 42$ and $t = 7$. After 7 minutes.

(c) The run from $t = 0$ to $t = 12$ is 12 ÷ 3 = 4 squares. The rise from $h = -72$ to $h = 0$ is 72 ÷ 36 = 2 squares. On this grid the line climbs half a square for each square across, even though its slope is 6 meters per minute: the slope is measured in units, not squares.

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