The Pythagorean theorem

In a right triangle, two sides give you the third. Square, add or subtract, then take the square root.

WORDS TO KNOW

Right triangle: a triangle with one 90° angle, often marked with a small square in the corner.

Hypotenuse: the side across from the right angle, called c. It is always the longest side.

Exact answer: a value kept with its root, such as $\sqrt{73}$, not the rounded 8.54.

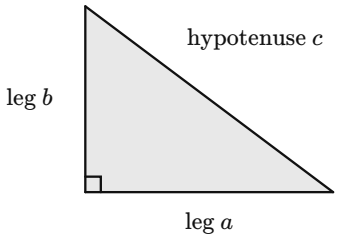
Legs: the two sides that meet at the right angle. We call them a and b.

Square root: $\sqrt{n}$ is the number that is not negative and whose square is n. For example, $\sqrt{49} = 7$.

KEY IDEAS

1. Leg squared plus leg squared equals hypotenuse squared

In every right triangle, the squares of the two legs add up to the square of the hypotenuse. Which leg is a and which is b does not matter. Which side is c matters a lot: it is the side across from the square corner.

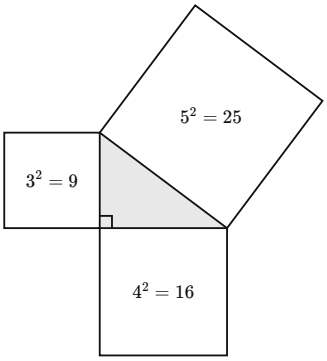


The Pythagorean theorem. In a right triangle with legs a and b and hypotenuse c,

$$a^2 + b^2 = c^2$$

2. The squares are real squares

Build a square outward on each side of a right triangle. The two squares on the legs together hold exactly as much area as the square on the hypotenuse. With legs 3 and 4, the squares hold 9 and 16, and the big square holds 25 = 9 + 16. That is also why the hypotenuse is the longest side: its square holds both of the others.



3. Hypotenuse: add. Leg: subtract.

To find the hypotenuse, add the squares of the legs, then take the square root:

$$c = \sqrt{a^2 + b^2}$$

To find a leg, take the square of the other leg away from the square of the hypotenuse:

$$a = \sqrt{c^2 - b^2}$$

Only the positive root counts, because a length is never negative.

Steps for a word problem: find the right angle and the reason it is there, draw the triangle, mark the hypotenuse, write the equation, solve, then answer the question that was asked.

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WATCH OUT

- **Find the hypotenuse before you write anything.** It is the side across from the right angle, however the triangle is turned. If the unknown is a leg, you subtract. Adding instead gives a "leg" longer than the hypotenuse, which is impossible.
- **The right angle must be given, or follow from the facts.** A wall standing on level ground, the corner of a rectangle, north and east: these make right angles. A corner that only looks square in a sketch is not enough. For a triangle with no right angle, the equation is false.
- **The theorem gives c^2, not c.** Do not stop at $c^2 = 73$. Take the square root, keep it exact, and round only at the very end.
- **Use one unit for all three sides.** Change meters to centimeters (or the other way) before you square anything.

WORKED EXAMPLES

Example 1. A right triangle has legs 3 and 8. Find the hypotenuse, exactly and to the nearest hundredth.

1. Both given sides are legs, so the unknown is the hypotenuse. Add the squares.
$$3^2 + 8^2 = c^2 \qquad 9 + 64 = c^2 \qquad 73 = c^2$$
2. No whole number squares to 73, so keep the root: $c = \sqrt{73}$.
3. Since $8^2 = 64$ and $9^2 = 81$, the root is between 8 and 9. A calculator gives about 8.54.

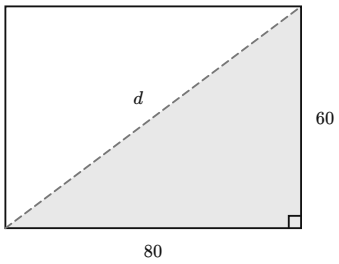
Answer: $c = \sqrt{73} \approx 8.54$. **Check:** 8.54 is longer than both legs, as a hypotenuse must be.

Example 2. In a right triangle, the hypotenuse is 17 and one leg is 8. Find the other leg b.

1. The 17 is the hypotenuse, so it stands alone: $8^2 + b^2 = 17^2$.
2. Work out the squares: $64 + b^2 = 289$.
3. Take 64 from both sides: $b^2 = 225$.
4. Since $15^2 = 225$, $b = 15$.

Answer: $b = 15$. **Check:** $64 + 225 = 289 = 17^2$, and 15 is shorter than the hypotenuse 17.

Example 3. A rectangular field is 80 m long and 60 m wide. How long is the straight path from one corner to the opposite corner, and how much shorter is it than walking along two sides?



1. Every corner of a rectangle is a right angle. So the two sides and the diagonal d make a right triangle, with d as the hypotenuse.
2. Add the squares of the legs.
$$d^2 = 80^2 + 60^2 = 6400 + 3600 = 10000$$
3. Since $100^2 = 10000$, $d = 100$ m.
4. The question also asks how much shorter. Along two sides is 80 + 60 = 140 m, and $140 - 100 = 40$ m.

Answer: the straight path is 100 m, which is 40 m shorter. **Check:** 100 is longer than each side but shorter than the two sides together.

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