Math 8 Unit 11 worksheet: The Pythagorean theorem (answer key)

ANSWERS AT A GLANCE

1. (a) $c = 37$ (b) $a = 28$ (c) $c = \sqrt{29} \approx 5.39$ (d) $b = \sqrt{85} \approx 9.22$ (e) $c = 1.5$ (f) $b = 4$
2. (a) $\sqrt{97}$ is between 9 and 10, nearer 10. (b) $\sqrt{119}$ is between 10 and 11, nearer 11.
3. (a) A leg can never be longer than the hypotenuse, and 23.3 > 20. (b) 16
4. (a) No: $7.5^2 + 7.5^2 = 112.5$, but $10^2 = 100$. (b) $\sqrt{43.75} = \frac{5\sqrt{7}}{2} \approx 6.61$
5. (a) $EF = 12$ (b) $RT = 60$ (c) $PR = \sqrt{45} = 3\sqrt{5} \approx 6.71$
6. 91 inches across
7. 52 cm

WORKED SOLUTIONS

1. Each right triangle has legs a and b and hypotenuse c. Two of the three sides are given. Find the missing side, exactly, and to the nearest hundredth when it is not a whole number.

(a) $a = 12,\ b = 35$ (b) $b = 45,\ c = 53$ (c) $a = 2,\ b = 5$

(d) $a = 6,\ c = 11$ (e) $a = 0.9,\ b = 1.2$ (f) $a = 0.9,\ c = 4.1$

ANSWER

(a) $c = 37$ (b) $a = 28$ (c) $c = \sqrt{29} \approx 5.39$ (d) $b = \sqrt{85} \approx 9.22$ (e) $c = 1.5$ (f) $b = 4$

SOLUTION

(a) Both legs are given, so add their squares: $c^2 = 12^2 + 35^2 = 144 + 1225 = 1369$, and $c = 37$ because $37^2 = 1369$.

(b) The hypotenuse is given, so the missing side is a leg: $a^2 = 53^2 - 45^2 = 2809 - 2025 = 784$, and $a = 28$.

(c) $c^2 = 2^2 + 5^2 = 4 + 25 = 29$, so $c = \sqrt{29} \approx 5.39$.

(d) $b^2 = 11^2 - 6^2 = 121 - 36 = 85$, so $b = \sqrt{85} \approx 9.22$.

(e) $c^2 = 0.9^2 + 1.2^2 = 0.81 + 1.44 = 2.25$, so $c = 1.5$.

(f) $b^2 = 4.1^2 - 0.9^2 = 16.81 - 0.81 = 16$, so $b = 4$.

In every part only the positive square root is a length.

2. Do not use a calculator.

(a) A right triangle has legs 4 and 9. Its hypotenuse lies between which two whole numbers? Which of the two is it nearer?

(b) A right triangle has hypotenuse 12 and one leg 5. The other leg lies between which two whole numbers? Which of the two is it nearer?

ANSWER

(a) $\sqrt{97}$ is between 9 and 10, nearer 10. (b) $\sqrt{119}$ is between 10 and 11, nearer 11.

SOLUTION

(a) $c^2 = 4^2 + 9^2 = 16 + 81 = 97$, so $c = \sqrt{97}$. Since $9^2 = 81$ and $10^2 = 100$, and 81 < 97 < 100, the hypotenuse is between 9 and 10. The number 97 is only 3 below 100 but 16 above 81, so $\sqrt{97}$ is nearer 10.

(b) The 12 is the hypotenuse, so subtract: $b^2 = 12^2 - 5^2 = 144 - 25 = 119$, and $b = \sqrt{119}$. Since $10^2 = 100$ and $11^2 = 121$, and 100 < 119 < 121, the leg is between 10 and 11, and much nearer 11, since 119 is only 2 below 121.

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3. A right triangle has a hypotenuse of 20 and one leg of 12. A student finds the other leg like this:

$$b^2 = 20^2 + 12^2 = 544, \qquad b = \sqrt{544} \approx 23.3$$

(a) Without working out the correct answer, explain how you can tell that 23.3 is wrong.

(b) Find the correct length of the other leg.

ANSWER

(a) A leg can never be longer than the hypotenuse, and 23.3 > 20. (b) 16

SOLUTION

(a) The hypotenuse is the longest side of a right triangle, because its square is the sum of the two other squares. A leg of 23.3 would be longer than the hypotenuse 20, so the answer must be wrong. The student added the squares as if the unknown were the hypotenuse.

(b) The 20 is the hypotenuse, so it stands alone: $12^2 + b^2 = 20^2$, so $b^2 = 400 - 144 = 256$ and $b = 16$. Check: $144 + 256 = 400 = 20^2$, and 16 < 20.

4. (a) Can a right triangle have a hypotenuse of 10 and two legs that are both 7.5? Explain using the Pythagorean theorem.

(b) A right triangle has a hypotenuse of 10 and one leg of 7.5. Find the other leg, exactly and to the nearest hundredth.

ANSWER

(a) No: $7.5^2 + 7.5^2 = 112.5$, but $10^2 = 100$. (b) $\sqrt{43.75} = \dfrac{5\sqrt{7}}{2} \approx 6.61$

SOLUTION

(a) In a right triangle the squares of the legs must add to the square of the hypotenuse. Here $7.5^2 + 7.5^2 = 56.25 + 56.25 = 112.5$, while $10^2 = 100$. The sum is too big, so no right triangle has these sides. Legs of 7.5 are too long to meet a hypotenuse of 10 at a right angle.

(b) $7.5^2 + b^2 = 10^2$, so $b^2 = 100 - 56.25 = 43.75$ and $b = \sqrt{43.75}$. Since $43.75 = \dfrac{175}{4}$ and $175 = 25 \cdot 7$, this is $\dfrac{5\sqrt{7}}{2} \approx 6.61$. As expected from (a), the second leg is shorter than 7.5.

5. No figure is given. Use the letters to decide which side is the hypotenuse, then find the side asked for. Give exact answers, and a decimal to the nearest hundredth when the answer is not a whole number.

(a) In $\triangle DEF$, $\angle E = 90^\circ$, $DE = 16$ and $DF = 20$. Find EF.

(b) In $\triangle RST$, $\angle R = 90^\circ$, $RS = 11$ and $ST = 61$. Find RT.

(c) In $\triangle PQR$, $\angle P = 90^\circ$, $PQ = 6$ and $QR = 9$. Find PR.

ANSWER

(a) $EF = 12$ (b) $RT = 60$ (c) $PR = \sqrt{45} = 3\sqrt{5} \approx 6.71$

SOLUTION

The hypotenuse is the side opposite the right angle, the one whose two letters do not include the right-angle vertex.

(a) The right angle is at E, so the hypotenuse is $\overline{DF} = 20$, and $\overline{DE}$ and $\overline{EF}$ are the legs: $16^2 + EF^2 = 20^2$, so $EF^2 = 400 - 256 = 144$ and $EF = 12$.

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(b) The right angle is at R, so the hypotenuse is $\overline{ST} = 61$: $11^2 + RT^2 = 61^2$, so $RT^2 = 3721 - 121 = 3600$ and $RT = 60$.

(c) The right angle is at P, so the hypotenuse is $\overline{QR} = 9$, not the unknown side: $6^2 + PR^2 = 9^2$, so $PR^2 = 81 - 36 = 45$ and $PR = \sqrt{45} = 3\sqrt{5} \approx 6.71$. Check: 6.71 is shorter than the hypotenuse 9.

6. A family is moving a thin round tabletop into a room. The doorway is a rectangle 35 inches wide and 84 inches tall. The tabletop is too wide to carry through upright, so it will be tilted to pass along the diagonal of the doorway. Ignoring the thickness of the top, what is the widest tabletop that can pass through this way?

ANSWER
91 inches across

SOLUTION
Every corner of a rectangle is a right angle, so the diagonal d of the doorway is the hypotenuse of a right triangle with legs 35 and 84:

$$\begin{aligned} 35^2 + 84^2 &= d^2 \\ 1225 + 7056 &= d^2 \\ 8281 &= d^2 \end{aligned}$$

Since $91^2 = 8281$, $d = 91$. The widest straight line across the doorway is its diagonal, so the tabletop can be at most 91 inches across.

Check: 91 is longer than both 35 and 84, as a hypotenuse must be.

7. A framed picture is to hang from a single nail. A wire will be fixed to the back of the frame at two points 48 cm apart, level with each other, and passed over the nail so that its two pieces, pulled straight, are the same length. The nail must end up 10 cm above the fixing points, directly above the point halfway between them, so a vertical line from the nail meets the line between the fixing points at a right angle. How long must the whole wire be?

ANSWER
52 cm

SOLUTION
The vertical line from the nail cuts the shape into two right triangles. Take one of them. Its right angle is where the vertical line meets the line between the fixing points, so its hypotenuse is one piece of wire, w. One leg is half the distance between the fixing points, 48 ÷ 2 = 24 cm, and the other leg is the height of the nail, 10 cm:

$$\begin{aligned} 24^2 + 10^2 &= w^2 \\ 576 + 100 &= w^2 \\ 676 &= w^2 \\ w &= 26 \end{aligned}$$

Each piece is 26 cm, and the question asks for the whole wire, two pieces: 2 × 26 = 52 cm.

Check: each piece, 26 cm, is longer than both legs, as a hypotenuse must be.

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