Precalculus Unit 1 test: Functions in depth: rate of change, end behavior and asymptotes

Name _____ Date _____ Period _____

12 questions, 51 points

1. Let $p(x) = x^5 - 16x$. (4 points)

(a) Factor $p(x)$ completely over the integers, and find all five of its zeros, real and non-real.

(b) Describe the end behavior of p in arrow notation, and give its range.

(c) Is p even, odd or neither? Show the work that decides it.

2. A polynomial p has degree 5 and a negative leading coefficient. Its formula is not given, but $p(-2) = 3$, $p(0) = -1$ and $p(2) = 4$. (4 points)

(a) Describe the end behavior of p in arrow notation.

(b) Explain why p has at least three real zeros, and say in which interval each one lies.

(c) Explain why the graph of p has at least two turning points. What is the greatest number of turning points it can have?

(d) Give the range of p.

3. Let $f(x) = \dfrac{x^3 - 7x - 6}{x^2 + x - 2}$. The numerator is 0 at $x = -1$. (4 points)

(a) Factor the numerator and the denominator completely, and give the domain of f.

(b) Find every hole and vertical asymptote, and say what the graph does on each side of each vertical asymptote.

(c) Find the line the graph follows far out, and say on which side of it the graph lies far to the right and far to the left.

(d) Find the intercepts and the sign of f on each interval, and sketch the graph.

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4. Let $f(x) = \dfrac{(x-2)(x+k)}{x(x-4)}$, where k is a constant. (4 points)

(a) For $k = -2$, find the asymptotes, what the graph does on each side of each vertical asymptote, the intercepts and the signs, and sketch the graph.

(b) Show that for every k the horizontal asymptote is $y = 1$, and that the graph can meet it only where $(k+2)x = 2k$.

(c) Find every value of k for which the graph never meets its horizontal asymptote. Explain each.

5. Let $f(x) = x^3 + 2x^2 - 6x$. (4 points)

(a) Find and simplify the difference quotient of f.

(b) Use it to show that the average rate of change of f on any interval $[-w, w]$, with $w > 0$, is $w^2 - 6$.

(c) Find the interval $[-w, w]$ on which the average rate of change of f is 10, and check it from the values of f at the ends.

6. Omar runs a print shop. Printing n posters of one design costs $C(n) = \dfrac{n^2}{100} + 2n + 200$ dollars. (4 points)

(a) Find and simplify $\dfrac{C(n+h) - C(n)}{h}$, and say what it measures, with units.

(b) An order grows from 100 to 200 posters. Use (a) to find the average cost of each extra poster.

(c) The order then grows by another 100 posters, to 300. Find the average cost of each of these extra posters, and how much more it is than in (b).

7. Let $f(x) = \dfrac{2x^3 + ax^2 + 9x + b}{x^2 + 4}$, where a and b are constants. (4 points)

(a) Find a so that the slant asymptote of f is $y = 2x + 1$.

(b) With that a, find b so that the graph meets its slant asymptote at $x = -3$, and give the point where they meet.

(c) Which side of the slant asymptote is the graph on far to the right?

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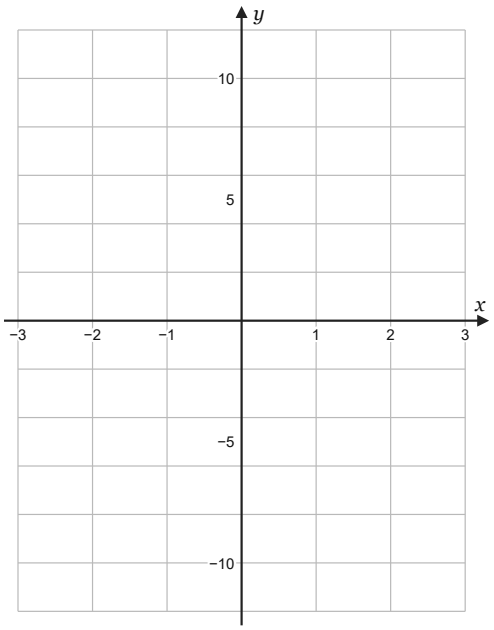
8. Let $p(x) = x^5 - 2x^4 - 5x^3 + 10x^2 + 4x - 8$. (5 points)

(a) Show that 2 is a zero of p, and factor $p(x)$ completely.

(b) Give each zero with its multiplicity and what the graph does there, the end behavior and the y-intercept.

(c) Give the sign of $p(x)$ on each interval between the zeros, and find $p\left(-\frac{3}{2}\right)$ and $p\left(\frac{3}{2}\right)$.

(d) Sketch the graph. List its peaks and valleys by interval, and explain why it can have no others.



9. Let $f(x) = x^4 - 3x$. (5 points)

(a) Simplify the right estimate $\frac{f(1+h) - f(1)}{h}$ and the centered average $\frac{f(1+h) - f(1-h)}{2h}$.

(b) Find the rate of change of f at $x = 1$. For small widths, do the right estimates lie above or below it, and the left estimates?

(c) With $h = 0.01$, find the right estimate and the centered average, and how far each is from the rate.

(d) Write the tangent line to the graph of f at $x = 1$.

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10. A 15-ohm resistor is wired in parallel with a second resistor of x ohms, where $x > 0$. The combined resistance is $R(x) = \dfrac{15x}{15 + x}$ ohms. (5 points)

(a) For which values of x is the combined resistance at least 12 ohms? Solve with a sign chart.

(b) Show that $R(x) < 15$ for every $x > 0$.

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11. Hiro surveys a cave passage that starts at the entrance and runs 800 m into a hillside. At x hundred meters along the passage, the floor is

$$E(x) = -\frac{1}{4}x(x-2)^2(x-7)$$

meters above the entrance, for $0 \le x \le 8$ (a negative value means below the entrance). (4 points)

(a) Where along the passage is the floor level with the entrance? At which of these places does the floor pass from above the entrance to below it?

(b) On which stretches of the passage is the floor above the entrance, and on which is it below?

(c) Explain why, between 200 m and 700 m in, the floor has exactly one peak. Find $E(5)$, $E(5.5)$ and $E(6)$, to one decimal place, and use them to name the 100 m stretch that contains this peak. Take the largest of the three values as your estimate of the peak's height.

(d) Find $E(8)$ and say what it means.

12. Naomi counts visitors at a county fair that is open for 6 hours. The total number of visitors who have come in, in tens, is modeled by $V(t) = -t^3 + bt^2 + ct$ for $0 \le t \le 6$ hours after opening, where b and c are constants. Visitors were arriving fastest 3 hours after opening, at 112 tens of visitors per hour. (4 points)

(a) Find b and c.

(b) Find where V is concave up and where it is concave down, and its point of inflection. What does the point of inflection say about the fair?

(c) Were visitors still arriving at closing time? Explain.

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