Precalculus Unit 1 worksheet: Concavity: when the rate of change itself changes

Name _____ Date _____ Period _____

1. Say where each function is concave up and where it is concave down, or say that it is neither. Name the result you used (quadratic, cubic, exponential or line), or reason from first differences if no result applies.

(a) $f(x) = 6(0.7)^x$

(b) $g(x) = 4x - 3x^2$

(c) $h(x) = 9 - 5x$

(d) $k(x) = \frac{1}{2}x^2 - 8x + 1$

(e) $p(x) = 2x^3 - 12x^2 + 7$

(f) $q(x) = -3(2)^x$

2. Jin takes a loaf of bread out of the oven and records the temperature at its center every 5 minutes.

minutes t	**0**	**5**	**10**	**15**	**20**	**25**
temperature (degrees Celsius)	98	74	56	43	34	28

(a) Find the average rate of change over each 5-minute step, in degrees per minute.

(b) Describe the temperature with "increasing" or "decreasing" and "concave up" or "concave down", and say in words what that means for the cooling.

(c) Jin predicts the bread will be at 20 degrees at $t = 30$. Is that prediction in line with the concavity of the table? Explain.

Free sample from math.wiki for classroom use. Do not post. math.wiki

3. During a storm, Sofia reads the total rainfall in her rain gauge. She reads it every 10 minutes at first, then every 20 minutes.

minutes t	**0**	**10**	**20**	**30**	**40**	**60**	**80**
total rain (mm)	0	2	7	15	22	30	32

(a) Find the average rate of rainfall over each step, in millimeters per minute.

(b) Where is the table concave up, where is it concave down, and between which readings does the concavity change? When was the rain falling hardest?

(c) Sofia looks at the first differences 2, 5, 8, 7, 8, 2 and says the rain picked up again after $t = 40$. Is she right? Explain.

4. Omar says that $g(x) = (x-3)^4 + 1$ has a point of inflection at $x = 3$, because its graph is flat there.

(a) Find the secant slopes on $[1,2]$, $[2,3]$, $[3,4]$ and $[4,5]$.

(b) What do the slopes suggest about the concavity of g, and is Omar right? What kind of point is (3, 1)?

(c) Give a function whose graph is flat at a point of inflection.

5. Let $f(x) = x^3 + 6x^2 + 9x - 7$.

(a) Find the point of inflection P of f, and the intervals where f is concave up and concave down.

(b) Show that $f(-2+t) = -9 - 3t + t^3$ for every number t. Use it to show that $f(-2+t) + f(-2-t)$ is the same number for every t, and say what that means for the graph and P.

(c) Use (b) to show that $m(-2-s, -2-t) = m(-2+t, -2+s)$ whenever $0 < t < s$. Explain why this makes the concavity to the left of P the opposite of the concavity to the right.

Free sample from math.wiki for classroom use. Do not post. math.wiki

6. Let $f(x) = x^3$.

(a) Show that the secant slope from a to b is $m(a,b) = a^2 + ab + b^2$.

(b) Prove that f is concave up on $[0, \infty)$: for every $0 \le a < b < c$, show $m(a,b) < m(b,c)$.

(c) Prove in the same way that f is concave down on $(-\infty, 0]$, and name the point of inflection.

Statements	Reasons

Free sample from math.wiki for classroom use. Do not post. math.wiki