Precalculus Unit 1 worksheet: Rates at an instant from shrinking intervals

Name _____ Date _____ Period _____

1. Let $f(x) = x^3 - 3x$.

(a) Simplify $\dfrac{f(2+h) - f(2)}{h}$ for $h \neq 0$.

(b) Use your answer to (a) to find the rate of change of f at $x = 2$. Explain.

(c) For each width 0.1, 0.01 and 0.001, find how far the right estimate and the left estimate are from the rate. For which of these widths are both estimates within 0.1 of the rate?

2. Let $f(x) = -2x^2 + 5x - 7$.

(a) Find the centered average of f at $x = -1$ with the wide width $h = 2$, and explain why this one number is already the rate of change of f at -1.

(b) Write the tangent line at $x = -1$ in the form $y = mx + b$, and use it to estimate $f(-0.9)$.

(c) Find $f(-0.9)$ exactly. By how much does the estimate in (b) miss?

3. Let

$$f(x) = \begin{cases} x^2, & x < 1 \\ x^2 + 1, & x \geq 1. \end{cases}$$

(a) Find the right estimates $\dfrac{f(1+h) - f(1)}{h}$ at $x = 1$ with $h = 0.1$ and $h = 0.01$.

(b) Find the left estimates at $x = 1$ with $h = -0.1$, $h = -0.01$ and $h = -0.001$.

(c) Ben looks only at part (a) and says the rate of change of f at 1 is 2. Is he right? Explain, using the definition of the rate at an instant.

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4. Maya heats water in an electric kettle and reads a thermometer at the times below. She skipped some readings, so the times are not evenly spaced.

t **(seconds)**	0	20	40	50	60	80
temperature (degrees Celsius)	22.0	31.0	39.2	43.0	46.6	53.2

(a) At $t = 50$, find a right estimate, a left estimate and a centered average of the rate at which the temperature rises, each with the narrowest width the table allows.

(b) Find the centered average at $t = 50$ with $h = 30$. What is your best estimate of the rate at $t = 50$?

(c) Explain why the table cannot give a centered average at $t = 40$ with $h = 10$. Find the narrowest centered average it does allow at $t = 40$.

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5. Diego rides a sprint on a velodrome. A laser timer records his distance from the start line, rounded to the nearest 0.1 meter.

t **(seconds)**	**3.90**	**4.00**	**4.01**	**4.10**
distance (meters)	29.4	30.2	30.3	31.0

(a) At $t = 4$, find the right estimates with widths 0.01 and 0.1, the left estimate with width 0.1, and the centered average with $h = 0.1$.

(b) Each reading can be off by up to 0.05 meter, so the difference of two readings can be off by up to 0.1 meter. At most, how far off can each of your four estimates be because of the rounding?

(c) Which estimate should Diego trust most, and about how fast was he riding at $t = 4$?

6. (a) Let $f(x) = px^2 + qx + c$ and $h > 0$. Show that the right estimate $\dfrac{f(a+h) - f(a)}{h}$ is $2pa + q + ph$ and the left estimate $\dfrac{f(a) - f(a-h)}{h}$ is $2pa + q - ph$.

(b) Leah knows only that g is a quadratic. At $x = 3$, with width 0.2, her right estimate of the rate is 7.3 and her left estimate is 6.5. Find the rate of change of g at 3 and the coefficient of x^2 in g.

(c) Find the coefficient of x in g.

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