Precalculus Unit 1 worksheet: Graphing rational functions

Name _____ Date _____ Period _____

1. Without sketching, give the sign ($+$ or $-$) of each function on each interval of its domain between its zeros and vertical asymptotes. Name any hole, and say whether it changes the sign.

(a) $\dfrac{(x+3)(x-1)^2}{x(x-4)}$

(b) $\dfrac{2-x}{(x+1)^2}$

(c) $\dfrac{3x^2+6x}{4-x^2}$

2. Let $f(x) = \dfrac{x^2-9}{x^3-4x}$.

(a) Factor, and give the domain, any holes, each vertical asymptote with what the graph does on each side of it, the horizontal asymptote, and the intercepts.

(b) Give the sign of f on each interval.

(c) Show that $f(-x) = -f(x)$, and say what this tells you about the graph. Then sketch it.

3. Let $g(x) = \dfrac{x^2-2x+1}{x+1}$.

(a) Find the vertical asymptote and what the graph does on each side of it.

(b) Use long division to find the line the graph follows far out. Far to the right, is the graph above or below that line? Far to the left? Does the graph ever meet the line?

(c) Find the intercepts and the sign on each interval, saying whether the graph crosses or touches the axis. Then sketch the graph.

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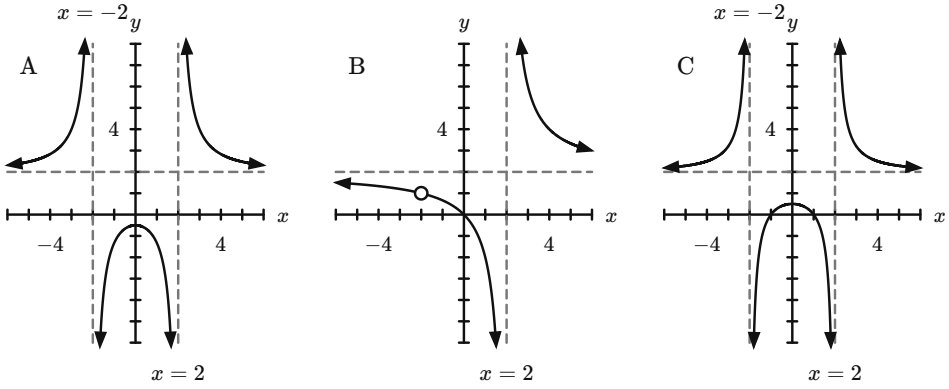
4. A rational function has the form $f(x) = 2x - 1 + \dfrac{c}{x-3}$ for a nonzero constant c, and its graph passes through the origin.

(a) Find c, and write f as a single fraction.

(b) Give the vertical asymptote and the slant asymptote, and find the other zero of f.

(c) Far to the right, is the graph above or below its slant asymptote? Explain.

5. Each of the graphs A, B and C below is the graph of one of these rules. Asymptotes are dashed, open circles are holes, and tick marks are 1 unit apart.

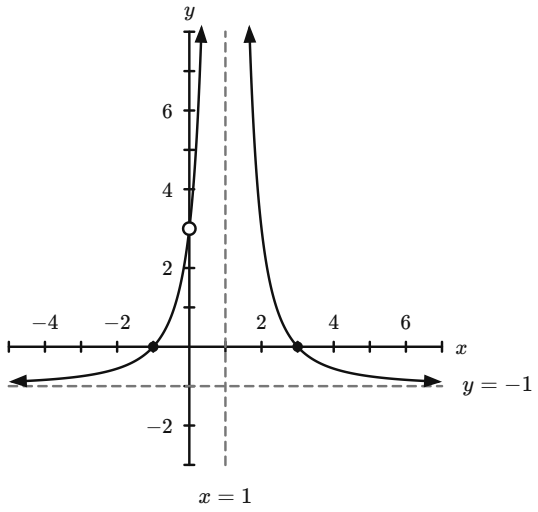


(1) $\dfrac{2x^2 - 2}{x^2 - 4}$ (2) $\dfrac{2x^2 + 2}{x^2 - 4}$ (3) $\dfrac{2x^2 + 4x}{x^2 - 4}$ (4) $\dfrac{2 - 2x^2}{4 - x^2}$ (5) $\dfrac{2x^2 - 4x}{x^2 - 4}$

One graph fits two of the rules, and one rule fits none of the graphs. Match each rule to its graph, or say that it has none. For each, name the features that decide it.

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6. The graph of $f(x) = \dfrac{-x^3 + 2x^2 + 3x}{x^3 - 2x^2 + x}$ is drawn below, with its asymptotes dashed and tick marks 1 unit apart.



(a) Factor, and find the hole. Explain why the graph has no y-intercept, even though it reaches the y-axis.

(b) Use the factors to explain why the graph goes up on both sides of $x = 1$, and why it crosses the axis at $x = -1$ and at $x = 3$.

(c) Show that the graph never meets its horizontal asymptote, and say whether it lies above or below it.

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