Precalculus Unit 7 worksheet: Law of cosines (answer key)

ANSWERS AT A GLANCE

1. (a) $c \approx 8.57$ (b) $a \approx 10.92$ (c) $b \approx 17.71$ (d) $c \approx 6.25$
2. (a) $A = (-1, \sqrt{3})$ (b) $c = 2\sqrt{7}$ (c) the law gives $c^2 = 28$ as well
3. (a) $\angle FHT \approx 106.07^\circ$ (b) $FT \approx 39.94$ cm
4. (a) $x = \sqrt{119}$ or $x = 13$ (b) $7 < x < \sqrt{119}$ or $13 < x < 17$ (c) $\sqrt{119} < x < 13$
5. $\cos B = -\dfrac{1}{24}$; the triangle is obtuse.
6. (a) Her A and B are almost equal, but $b = 13$ is much longer than $a = 7$; the longer side must be across from the larger angle. (b) $A \approx 30.50^\circ$, $B \approx 109.50^\circ$

WORKED SOLUTIONS

1. Find the side named in each triangle ABC. Use a calculator and round to two decimal places.

(a) $a = 9$, $b = 13$, $C = 41^\circ$; find c.

(b) $b = 5.5$, $c = 8.2$, $A = 104^\circ$; find a.

(c) $a = 17$, $c = 12$, $B = 73^\circ$; find b.

(d) $a = 3.6$, $b = 2.9$, $C = 148^\circ$; find c.

ANSWER

(a) $c \approx 8.57$ (b) $a \approx 10.92$ (c) $b \approx 17.71$ (d) $c \approx 6.25$

SOLUTION

In each, the given angle is between the two given sides, so it goes with the side across from it.

(a) $c^2 = 9^2 + 13^2 - 2(9)(13)\cos 41^\circ = 250 - 234\cos 41^\circ \approx 73.398$, so $c \approx 8.57$.

(b) $a^2 = 5.5^2 + 8.2^2 - 2(5.5)(8.2)\cos 104^\circ = 97.49 - 90.2\cos 104^\circ \approx 119.311$, so $a \approx 10.92$. The cosine is negative, so the term is added and a is longer than either given side.

(c) $b^2 = 17^2 + 12^2 - 2(17)(12)\cos 73^\circ = 433 - 408\cos 73^\circ \approx 313.712$, so $b \approx 17.71$.

(d) $c^2 = 3.6^2 + 2.9^2 - 2(3.6)(2.9)\cos 148^\circ = 21.37 - 20.88\cos 148^\circ \approx 39.077$, so $c \approx 6.25$.

2. Triangle ABC is placed on the coordinate plane with C at the origin and $B = (4, 0)$ on the positive x-axis. Side $\overline{CA}$ has length 2, and angle $C = 120^\circ$ is measured from the positive x-axis.

(a) Find the exact coordinates of A.

(b) Use the distance formula to find $c = AB$ exactly.

(c) Check your answer to (b) with the law of cosines.

ANSWER

(a) $A = (-1, \sqrt{3})$ (b) $c = 2\sqrt{7}$ (c) the law gives $c^2 = 28$ as well

SOLUTION

(a) The point at distance b from the origin in the direction of angle C is $(b\cos C,\ b\sin C)$:

$$A = (2\cos 120^\circ,\ 2\sin 120^\circ) = \left(2 \cdot \left(-\frac{1}{2}\right),\ 2 \cdot \tfrac{\sqrt{3}}{2}\right) = (-1, \sqrt{3}).$$

(b) With $B = (4, 0)$,

$$c^2 = (-1 - 4)^2 + (\sqrt{3} - 0)^2 = 25 + 3 = 28,$$

so $c = 2\sqrt{7}$.

(c) $c^2 = a^2 + b^2 - 2ab\cos C = 16 + 4 - 2(4)(2)\left(-\dfrac{1}{2}\right) = 16 + 4 + 8 = 28$, the same value.

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3. A laptop is hinged at H. The hinge is 24 cm from the front edge F of the keyboard and 21 cm from the top edge T of the screen, and the distance from F to T is 36 cm. Use a calculator.

(a) Find the opening angle $\angle FHT$ between the keyboard and the screen, to the nearest hundredth of a degree.

(b) The screen is tilted back until $\angle FHT = 125^\circ$. How far is the top edge of the screen from the front edge now? Round to two decimal places.

ANSWER

(a) $\angle FHT \approx 106.07^\circ$ (b) $FT \approx 39.94$ cm

SOLUTION

(a) Triangle FHT has all three sides, and the angle at H is across from $FT = 36$:

$$\cos H = \frac{24^2 + 21^2 - 36^2}{2(24)(21)} = \frac{576 + 441 - 1296}{1008} = -\frac{279}{1008} \approx -0.2768,$$

so $\angle FHT \approx 106.07^\circ$. The negative cosine says the screen is opened past a right angle.

(b) Now the two sides and the angle between them are known (SAS):

$$FT^2 = 24^2 + 21^2 - 2(24)(21)\cos 125^\circ = 1017 - 1008\cos 125^\circ \approx 1595.17,$$

so $FT \approx 39.94$ cm. A wider angle gives a longer third side, as it should.

4. A triangle has sides 5, 12 and x. Find, exactly, every value of x for which the triangle is (a) right, (b) obtuse, (c) acute.

ANSWER

(a) $x = \sqrt{119}$ or $x = 13$ (b) $7 < x < \sqrt{119}$ or $13 < x < 17$ (c) $\sqrt{119} < x < 13$

SOLUTION

First, the sides must make a triangle: $12 - 5 < x < 12 + 5$, so $7 < x < 17$.

Compare the square of the longest side with the sum of the other two squares.

If $x \geq 12$, then x is the longest side. The triangle is right when $x^2 = 5^2 + 12^2 = 169$, so $x = 13$; obtuse when $x^2 > 169$, so $13 < x < 17$; acute when $12 \leq x < 13$.

If $x < 12$, then 12 is the longest side. The triangle is right when $144 = 25 + x^2$, so $x = \sqrt{119} \approx 10.91$; obtuse when $144 > 25 + x^2$, so $7 < x < \sqrt{119}$; acute when $\sqrt{119} < x < 12$.

Together: (a) right for $x = \sqrt{119}$ or $x = 13$; (b) obtuse for $7 < x < \sqrt{119}$ or $13 < x < 17$; (c) acute for $\sqrt{119} < x < 13$.

5. Triangle ABC has $a = 9$, $b = 10$ and $c = 4$. Without a calculator, find the cosine of its largest angle as a fraction in lowest terms, name the angle, and say whether the triangle is acute, right or obtuse.

ANSWER

$\cos B = -\dfrac{1}{24}$; the triangle is obtuse.

SOLUTION

The longest side is $b = 10$, so the largest angle is B, the angle between a and c:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{81 + 16 - 100}{2(9)(4)} = \frac{-3}{72} = -\frac{1}{24}.$$

The cosine is negative, so B is obtuse. Check by squares: $b^2 = 100$ and $a^2 + c^2 = 97$.

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6. Priya solves the triangle with $a = 7$, $b = 13$ and $C = 40^\circ$. She finds $c \approx 8.86$ correctly. Then she writes $\sin B = \dfrac{13 \sin 40^\circ}{c}$, gets $B \approx 70.50^\circ$, and finishes with $A = 180^\circ - 40^\circ - 70.50^\circ = 69.50^\circ$.

(a) Without computing any new angle, explain why her angles cannot be right.

(b) Find the correct A and B to the nearest hundredth of a degree. Use a calculator.

ANSWER

(a) Her A and B are almost equal, but $b = 13$ is much longer than $a = 7$; the longer side must be across from the larger angle. (b) $A \approx 30.50^\circ$, $B \approx 109.50^\circ$

SOLUTION

(a) In a triangle the larger side is across from the larger angle, and nearly equal angles have nearly equal sides across from them. Her $A \approx 69.50^\circ$ and $B \approx 70.50^\circ$ differ by 1°, yet $b = 13$ is almost twice $a = 7$. Her mistake: B is across from the longer given side, so it is the angle that may be obtuse, and the inverse sine returned its supplement.

(b) First, $c^2 = 7^2 + 13^2 - 2(7)(13)\cos 40^\circ = 218 - 182\cos 40^\circ \approx 78.580$, so $c \approx 8.8645$ (keep the full value). The safe angle is A, across from the shorter given side 7:

$$\sin A = \frac{7 \sin 40^\circ}{c} \approx 0.5076, \qquad A \approx 30.50^\circ.$$

Then $B = 180^\circ - 40^\circ - 30.50^\circ = 109.50^\circ$. **Check** by cosines: $\cos B = \dfrac{7^2 + c^2 - 13^2}{2(7)(c)} \approx -0.3338$, which is negative, and $B \approx 109.50^\circ$. Her 70.50° is the supplement, 180° − 109.50°.

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