Shifting and rescaling data

Add the same fee to every order, or change every length from centimeters to millimeters. What happens to the mean, the median and the spread, without redoing the data?

WORDS TO KNOW

Shift: add (or take away) the same number c from every value.

Center and position: the mean, the median, the quartiles, the minimum and the maximum.

Rule $y = a + bx$: each old value x becomes y; multiply by b, then add a.

Rescale: multiply every value by the same positive number k, as in a change of units.

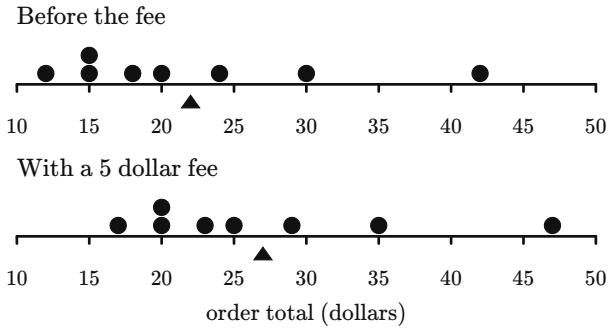
Spread: the range, the IQR, the standard deviation and the variance.

KEY IDEAS

1. Adding a constant slides the data

Every value moves the same distance, so the whole plot slides with every gap kept. The distances from the mean do not change.

> Add c to every value:
>
> center and position (mean, median, quartiles, min, max) all **add** c;
>
> spread (range, IQR, standard deviation, variance) **does not change**.



2. Multiplying by $k > 0$ stretches the data

Every value, and every distance between values, is multiplied by k. The shape stays the same.

> Multiply every value by $k > 0$:
>
> center, position and spread are all **multiplied by** k;
>
> the variance is multiplied by k^2, because it is made of squared distances.

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3. Both at once: $y = a + bx$

Write the change as one rule first, then apply the two results.

$$\bar{y} = a + b\bar{x} \qquad \sigma_y = b\,\sigma_x \qquad \sigma_y^2 = b^2\sigma_x^2$$

Center and position go through the whole rule. Spread feels only the multiplier b.

Summary	**Add** c	**Multiply by** k	$y = a + bx$
mean, median, quartiles	$+c$	$\times k$	$a + b$(old)
range, IQR, standard deviation	same	$\times k$	$\times b$
variance	same	$\times k^2$	$\times b^2$

WATCH OUT

- **The variance gets b^2, not b.** The standard deviation gets b.
- **The added number never touches a spread.** It cancels whenever one value is taken from another.
- **The order of steps matters.** Adding a 2 dollar fee and then raising by 25% is $1.25(x + 2) = 1.25x + 2.5$, not $1.25x + 2$.
- **The rule must be the same for every value.** A fee added to only some orders is not a shift, so work from the data again.

WORKED EXAMPLES

Example 1. A bathroom scale reads 0.4 kg too low. Its six weighings have mean 12.6 kg, median 12.3 kg, standard deviation 0.9 kg and IQR 1.4 kg. Give the corrected summaries.

1. The fix adds 0.4 to every reading: $y = x + 0.4$.
2. Center moves: mean 12.6 + 0.4 = 13.0 kg, median 12.3 + 0.4 = 12.7 kg.
3. Spread stays: standard deviation 0.9 kg, IQR 1.4 kg.

Answer: mean 13.0 kg, median 12.7 kg, standard deviation 0.9 kg, IQR 1.4 kg.

Example 2. Seedling heights have mean 45 cm, median 42 cm, standard deviation 8 cm and variance 64 square cm. Give all four in millimeters.

1. 1 cm = 10 mm, so every height is multiplied by $k = 10$. Nothing is added.
2. Mean 450 mm, median 420 mm, standard deviation 10 × 8 = 80 mm.
3. Variance: multiply by $k^2 = 100$, so 64 × 100 = 6400 square mm. Check: $80^2 = 6400$.

Answer: mean 450 mm, median 420 mm, standard deviation 80 mm, variance 6400 square mm.

Example 3. A taxi charges 3 dollars plus 2 dollars per mile. One day the trips had mean 6 miles, median 5 miles, standard deviation 2.5 miles and IQR 4 miles. Find the same summaries for the fares.

1. A trip of x miles costs $y = 3 + 2x$ dollars, so $a = 3$ and $b = 2$.
2. Center goes through the whole rule: mean 3 + 2 × 6 = 15 dollars, median 3 + 2 × 5 = 13 dollars.
3. Spread feels only the 2: standard deviation 2 × 2.5 = 5 dollars, IQR 2 × 4 = 8 dollars.

Answer: mean 15 dollars, median 13 dollars, standard deviation 5 dollars, IQR 8 dollars. The 3 dollar charge is the same on every trip, so it moves the fares but does not spread them.

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