Computing a standard deviation by hand

How far do the values in a list typically sit from their mean, in the data's own unit? The standard deviation answers that. It has one version for a whole group and one for a sample.

WORDS TO KNOW

Population: every member of the group you want to describe. Its size is N, its mean μ (mu), its standard deviation σ (sigma).

Deviation: a value minus the mean. The deviations always add to 0.

Frequency table: a table that lists each different value once with its count f.

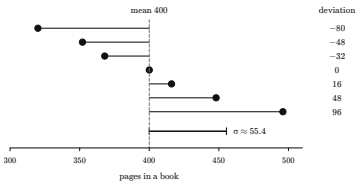
Sample: some members that stand in for a bigger group. Its size is n, its mean $\bar{x}$, its standard deviation s.

$\sum$ **(capital sigma):** "add this up over every value".

KEY IDEAS

1. The population standard deviation

Use a table with three columns: value, deviation, squared deviation. Keep the mean exact and round only at the end.



$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

Find the mean, the deviations (check they add to 0), square them, add, divide by N, then take the square root.

2. Frequency tables

A value that appears f times counts f times. So multiply each squared deviation by its count before adding.

Divide by the total of the counts, not by the number of rows.

3. The sample standard deviation

Same table, but divide by one less than the count.

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

The deviations are measured from the sample's own mean, which sits right in the middle of the sample. That makes the squares add to a little less than they would from the true mean of the whole group. Dividing by $n - 1$ makes the answer a little bigger to make up for it.

4. Which one to use, and sense checks

The question decides, not the numbers. If the values are the whole group you describe, use σ. If they stand in for a bigger group, use s.

σ is never larger than the largest distance from the mean.

$\sigma = 0$ only when every value is the same.

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WATCH OUT

- **Do not round the mean first.** A rounded mean makes the deviations stop adding to 0, and the answer drifts.
- **Take the square root.** The number before the root is the variance, in square units.
- **In a frequency table, divide by the number of values,** not the number of rows. For a sample, that is the number of values minus 1.
- **A sample needs at least two values.** With one value, $n - 1$ is 0, and you cannot divide by 0.

WORKED EXAMPLES

Example 1. The six members of a chess club are aged 12, 13, 15, 16, 18 and 22. Find the standard deviation of the club's ages.

1. The six are the whole club, so they are a population. The ages add to 96, so $\mu = 96 \div 6 = 16$ years.
2. Deviations: −4, −3, −1, 0, 2, 6. They add to 0.
3. Squares: 16, 9, 1, 0, 4, 36. They add to 66.
4. Divide by $N = 6$ and take the root.

$$\sigma = \sqrt{\frac{66}{6}} = \sqrt{11} \approx 3.32 \text{ years}$$

Answer: $\sigma \approx 3.32$ years. **Check:** the largest distance from the mean is 6, and 3.32 is less than 6.

Example 2. All 20 students in a class said how many siblings they have. Find the standard deviation.

siblings x	**0**	**1**	**2**	**3**
students f	2	8	6	4

1. The whole class is counted, so this is a population with $N = 20$.
2. The total of siblings is $0 \cdot 2 + 1 \cdot 8 + 2 \cdot 6 + 3 \cdot 4 = 32$, so $\mu = 32 \div 20 = 1.6$.
3. Deviations −1.6, −0.6, 0.4, 1.4. Squares 2.56, 0.36, 0.16, 1.96.
4. Times the counts: 5.12, 2.88, 0.96, 7.84. They add to 16.8.
5. Divide by the 20 students, not the 4 rows, and take the root.

$$\sigma = \sqrt{\frac{16.8}{20}} = \sqrt{0.84} \approx 0.92$$

Answer: $\sigma \approx 0.92$ siblings.

Example 3. A farmer picks 5 tomatoes at random from a crop of 400. They weigh 82, 90, 95, 99 and 104 grams. Find the right standard deviation.

1. The five stand in for all 400, so they are a sample with $n = 5$.
2. The masses add to 470, so $\bar{x} = 470 \div 5 = 94$ g.
3. Deviations: −12, −4, 1, 5, 10. They add to 0. Squares: 144, 16, 1, 25, 100, adding to 286.
4. Divide by $n - 1 = 4$ and take the root.

$$s = \sqrt{\frac{286}{4}} = \sqrt{71.5} \approx 8.46 \text{ g}$$

Answer: $s \approx 8.46$ g. Dividing by 5 instead would give $\sqrt{57.2} \approx 7.56$ g, which is too small for a sample.

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