Quartiles, range and IQR

The median tells you the middle, but not how much the values vary. Cut the sorted data into four parts with the same number of values, and the cut points show how spread out the data is.

WORDS TO KNOW

Lower half, upper half: the values before the median and the values after it, in the sorted list.

First quartile, Q_1: the median of the lower half.

Third quartile, Q_3: the median of the upper half.

Five-number summary: the minimum, Q_1, the median, Q_3 and the maximum, in that order.

Range: maximum minus minimum.

Interquartile range (IQR): $Q_3 - Q_1$, the width of the middle half of the data.

KEY IDEAS

1. Split at the median, then split each half

Sort the data and find the median.

Even count: the lower half is the first half of the values, and the upper half is the rest.

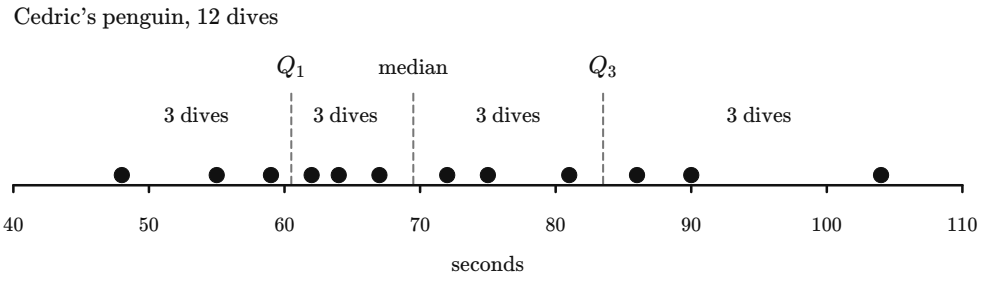
Odd count: the median is left out of both halves.

Q_1 is the median of the lower half, and Q_3 is the median of the upper half.

2. Four parts with equal counts, not equal widths

The quartiles cut the data into four quarters that hold about the same number of values. A narrow quarter means the values are crowded together; a wide one means they are spread out.

$$\text{minimum} \le Q_1 \le \text{median} \le Q_3 \le \text{maximum}$$



3. Two measures of spread

$$\text{range} = \text{maximum} - \text{minimum} \qquad \text{IQR} = Q_3 - Q_1$$

The range uses only the two most extreme values. The IQR is the spread of the ordinary values in the middle. Both are in the units of the data, and the IQR is never more than the range.

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WATCH OUT

- **Sort before you split.** The halves are halves of the sorted list.
- **With an odd count, leave the median out.** For 3, 5, 8, 10, 13, 15, 20, the lower half is 3, 5, 8, so $Q_1 = 5$. (Some books keep the median in; this lesson does not.)
- Q_1 **is not a quarter of the way along the number line.** It is a quarter of the way through the sorted values.
- **One extreme value can make the range huge.** With six or more values, making the largest value even larger does not move the IQR.

WORKED EXAMPLES

Example 1. Nine sunflower seedlings are 42, 24, 64, 33, 48, 67, 31, 62 and 36 cm tall. Find the five-number summary, the range and the IQR.

1. Sort: 24, 31, 33, 36, 42, 48, 62, 64, 67.
2. $n = 9$, so the median is the 5th value, 42. Leave it out of both halves.
3. Lower half: 24, 31, 33, 36. Its median is halfway between 31 and 33, so $Q_1 = 32$.
4. Upper half: 48, 62, 64, 67. Its median is halfway between 62 and 64, so $Q_3 = 63$.
5. Range: 67 − 24 = 43 cm. IQR: 63 − 32 = 31 cm.

Answer: five-number summary 24, 32, 42, 63, 67 cm; range 43 cm; IQR 31 cm.

Example 2. A team scored 44, 20, 57, 31, 60, 40, 24, 55, 34 and 42 points in ten games. Find the five-number summary and the IQR. Then the 60 is changed to 100. What happens to the range and the IQR?

1. Sort: 20, 24, 31, 34, 40, 42, 44, 55, 57, 60.
2. $n = 10$, so the middle values are the 5th and 6th, 40 and 42. The median is 41.
3. The count is even, so nothing is left out. Lower half: 20, 24, 31, 34, 40, so $Q_1 = 31$. Upper half: 42, 44, 55, 57, 60, so $Q_3 = 55$.
4. IQR: 55 − 31 = 24. Range: 60 − 20 = 40.
5. With 100 in place of 60, the sorted order stays the same, and the new value is still the largest. Q_1 and Q_3 do not use it.

Answer: five-number summary 20, 31, 41, 55, 60; IQR 24 points. After the change the range grows to 100 − 20 = 80, but the IQR stays 24.

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