Resistant and non-resistant measures

One water bill jumps because of a leak. Which summaries of the bills hardly move, and which get pushed a long way?

WORDS TO KNOW

Extreme value: the largest or smallest value in a data set, especially one far from the rest.

Not resistant: one extreme value, moved far enough, can push it as far as you like.

Resistant: one or a few extreme values barely move it, however far out they sit.

Center, spread: a typical value (mean, median), and how far apart the values are (range, IQR, MAD).

KEY IDEAS

1. Two lists to remember

Resistant: the median, the quartiles Q_1 and Q_3, and the IQR.

Not resistant: the mean, the range and the MAD.

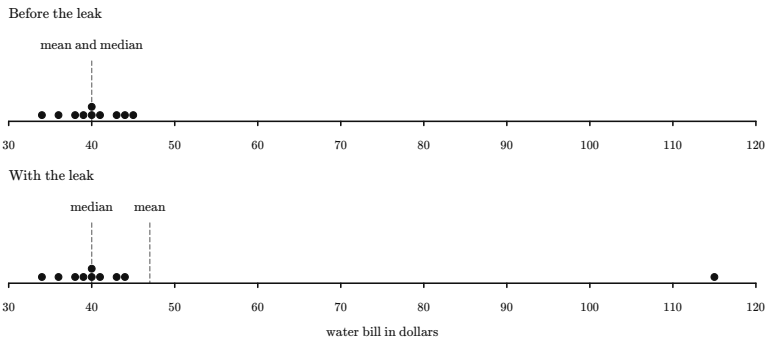
2. Positions or sizes

The median and the quartiles care about **positions** in the sorted list. If the largest value gets larger, it is still last, so every value keeps its place. The mean, the range and the MAD use the **sizes** of the values, so a far value pulls them.

Make the largest value larger (or the smallest smaller):

the median does not change (with at least 3 values), and Q_1, Q_3 and the IQR do not change (with at least 6 values);

the mean moves by the change divided by the number of values, and the range grows by the whole change.



3. Pick the summary that fits the question

For a typical value when the data has an extreme value, use the median with the IQR. But a total is always the mean times the count, so for a total you need the mean.

WATCH OUT

- **Resistant is not frozen.** Adding or removing a value shifts the halves, so the median and quartiles can move a little.
- **Resistant does not catch mistakes.** The median ignores a typing slip, so it cannot warn you of one. Use the 1.5 IQR rule too.
- **Very small data sets.** With fewer than 6 values, Q_1 and Q_3 can use the end values, so the IQR can move.

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WORKED EXAMPLES

Example 1. Seven quiz scores are 22, 24, 25, 26, 27, 28, 30. The 30 is typed as 100 by mistake. Find the median and the mean both ways.

1. Correct list: the median is the 4th value, 26. The total is 182, so the mean is 182 ÷ 7 = 26.
2. As typed, the 100 is still last. Every value keeps its place, so the median is still 26.
3. The total grows by 100 − 30 = 70, to 252. The mean becomes 252 ÷ 7 = 36.

Answer: the median stays 26, but the mean jumps from 26 to 36, more than any real score.

Example 2. Eight bike rides were 22, 25, 27, 28, 30, 31, 33 and 36 km. Find the range and the IQR. Then the 36 km ride is changed to 86 km. Find them again.

1. Range: 36 − 22 = 14 km.
2. Halves: 22, 25, 27, 28 and 30, 31, 33, 36. So $Q_1 = 26$, $Q_3 = 32$, and the IQR is 6 km.
3. With 86: the range is 86 − 22 = 64 km.
4. The upper half is 30, 31, 33, 86. Its middle two are still 31 and 33, so $Q_3 = 32$ and the IQR is still 6 km.

Answer: the range grows from 14 to 64 km, but the IQR stays 6 km.

Example 3. Six players are aged 9, 10, 10, 11, 12, 14. Their coach, aged 39, joins them. Find the mean and the median before and after.

1. Before: the total is 66, so the mean is 66 ÷ 6 = 11. The median is halfway between 10 and 11: 10.5.
2. After: the total is 66 + 39 = 105, so the mean is 105 ÷ 7 = 15.
3. Sorted: 9, 10, 10, 11, 12, 14, 39. The median is the 4th value, 11.

Answer: the mean moves from 11 to 15, older than every player. The median only moves from 10.5 to 11, the next value up.

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