Statistics Unit 1 quiz 2: Quartiles and the five number summary, Computing a standard deviation, sample and population, Interpreting a standard deviation (answer key)

7 questions, 21 points

ANSWERS AT A GLANCE

1. $Q_1 = 34$ s, median 45 s, $Q_3 = 56$ s
2. Isabel's. Jin's summary is 10, 14.5, 19, 26, 32 (Q_1 and Q_3 differ); Arjun's is 10, 14, 18, 26, 32 (the median and Q_3 differ).
3. (a) $Q_1 = 6$, $Q_3 = 18$ minutes. (b) Jin's IQR 9.5, the lesson's IQR 12 minutes; 2.5 minutes larger.
4. (a) s, because the five students stand in for the whole class. (b) $s = \sqrt{2150} \approx 46.37$ milliseconds
5. (a) The bakery's croissants typically weigh about 3 g more or less than their mean of 62 g. (b) 3 standard deviations below the mean. (c) From 56 g to 68 g; yes, 67 g is sold.
6. The fourth mass is 69 grams; $s = \sqrt{\frac{100}{3}} \approx 5.77$ grams.
7. (a) Hot line: mean 5 min, standard deviation ≈ 1.41 min. Salad line: mean 5 min, standard deviation ≈ 4.98 min. (b) Both waits average 5 minutes; the salad line's wait is far less predictable. (c) The hot line.

WORKED SOLUTIONS

1. Nine students in Hiro's class did a wall sit, back flat against the wall and knees bent, and timed how long each one held it, in seconds: 38, 52, 26, 65, 45, 41, 60, 30, 49. Find Q_1, the median and Q_3. (2 points)

ANSWER

$Q_1 = 34$ s, median 45 s, $Q_3 = 56$ s

SOLUTION

Sorted: 26, 30, 38, 41, 45, 49, 52, 60, 65. With 9 values the median is the 5th, 45 seconds, and it is left out of both halves. Lower half 26, 30, 38, 41: $Q_1 = \frac{30 + 38}{2} = 34$ seconds. Upper half 49, 52, 60, 65: $Q_3 = \frac{52 + 60}{2} = 56$ seconds.

2. Isabel, Jin and Arjun each counted how many lines they wrote in a journal on eight days.

Isabel: 24, 10, 18, 32, 12, 26, 16, 20

Jin: 20, 32, 14, 27, 10, 18, 25, 15

Arjun: 10, 16, 12, 22, 16, 20, 30, 32

Exactly one of the three lists has the five-number summary 10, 14, 19, 25, 32. Which one? For each of the other two lists, find its five-number summary and name the numbers in it that differ. (3 points)

ANSWER

Isabel's. Jin's summary is 10, 14.5, 19, 26, 32 (Q_1 and Q_3 differ); Arjun's is 10, 14, 18, 26, 32 (the median and Q_3 differ).

SOLUTION

Each list has 8 values, so the median is halfway between the 4th and 5th sorted values, and each half of four has its quartile halfway between its 2nd and 3rd values.

Isabel, sorted: 10, 12, 16, 18, 20, 24, 26, 32. Median $\frac{18 + 20}{2} = 19$, $Q_1 = \frac{12 + 16}{2} = 14$, $Q_3 = \frac{24 + 26}{2} = 25$. Summary 10, 14, 19, 25, 32: this is the list.

Jin, sorted: 10, 14, 15, 18, 20, 25, 27, 32. Median $\frac{18 + 20}{2} = 19$, $Q_1 = \frac{14 + 15}{2} = 14.5$, $Q_3 = \frac{25 + 27}{2} = 26$. Summary 10, 14.5, 19, 26, 32: Q_1 and Q_3 differ. (Reading the 2nd and 6th sorted values, 14 and 25, as the quartiles is the slip that makes Jin's list look right.)

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Arjun, sorted: 10, 12, 16, 16, 20, 22, 30, 32. Median $\frac{16+20}{2} = 18$, $Q_1 = \frac{12+16}{2} = 14$, $Q_3 = \frac{22+30}{2} = 26$. Summary 10, 14, 18, 26, 32: the median and Q_3 differ. (Working on the list without sorting it gives 14, 19, 25, which is why it can look right.)

SCORING

1 point: Isabel's list sorted, with median 19, $Q_1 = 14$, $Q_3 = 25$, named as the match. 1 point: Jin's summary 10, 14.5, 19, 26, 32, with Q_1 and Q_3 named as different. 1 point: Arjun's list sorted first, summary 10, 14, 18, 26, 32, with the median and Q_3 named as different.

3. Jin timed how many minutes each of 11 classmates took to finish a logic puzzle. Sorted, the times are 3, 5, 6, 8, 9, 12, 14, 15, 18, 20, 23.

Jin kept the median in both halves and got $Q_1 = 7$ and $Q_3 = 16.5$. (3 points)

(a) Find Q_1 and Q_3 by this lesson's rule.

(b) Find the IQR by Jin's way and by this lesson's rule. How much larger is the lesson's IQR?

ANSWER

(a) $Q_1 = 6$, $Q_3 = 18$ minutes. (b) Jin's IQR 9.5, the lesson's IQR 12 minutes; 2.5 minutes larger.

SOLUTION

(a) With 11 values the median is the 6th, 12. This lesson leaves it out of both halves, so the lower half is 3, 5, 6, 8, 9 and the upper half 14, 15, 18, 20, 23. Each has five values, so its median is its third: $Q_1 = 6$ and $Q_3 = 18$ minutes.

(b) Jin's IQR is 16.5 − 7 = 9.5 minutes. The lesson's is 18 − 6 = 12 minutes, which is 12 − 9.5 = 2.5 minutes larger.

SCORING

1 point: lower half 3, 5, 6, 8, 9 and upper half 14, 15, 18, 20, 23, with 12 left out. 1 point: $Q_1 = 6$ and $Q_3 = 18$. 1 point: IQRs 9.5 and 12, difference 2.5.

4. A class has 30 students. To estimate the spread of the class's reaction times, Mateo tests 5 students chosen at random. Their times are 180, 210, 250, 260 and 300 milliseconds. (3 points)

(a) Should Mateo use σ (divide by N) or s (divide by $n - 1$)? Say why.

(b) Find that standard deviation, to two decimal places.

ANSWER

(a) s, because the five students stand in for the whole class. (b) $s = \sqrt{2150} \approx 46.37$ milliseconds

SOLUTION

(a) Mateo wants to learn about all 30 students from 5 of them, so the five are a sample: use s and divide by $n - 1 = 4$.

(b) The mean is $\bar{x} = \frac{1200}{5} = 240$ ms. The deviations −60, −30, 10, 20, 60 add to 0, and their squares 3600, 900, 100, 400, 3600 add to 8600. So $s^2 = \frac{8600}{4} = 2150$ and $s = \sqrt{2150} \approx 46.37$ milliseconds.

SCORING

1 point: s, with the reason that the five stand in for the class. 1 point: the mean 240 and the sum of squared deviations 8600. 1 point: divides by 4 and gets $s \approx 46.37$ ms.

5. A bakery's croissants have a mean weight of 62 g and a standard deviation of 3 g. (3 points)

(a) Write one sentence interpreting the 3 g.

(b) One croissant weighs 53 g. How many standard deviations from the mean is it, and on which side?

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(c) The bakery sells only croissants within 2 standard deviations of the mean. Give the range of weights it sells, and say whether a 67 g croissant is sold.

ANSWER

(a) The bakery's croissants typically weigh about 3 g more or less than their mean of 62 g. (b) 3 standard deviations below the mean. (c) From 56 g to 68 g; yes, 67 g is sold.

SOLUTION

(a) The bakery's croissants typically weigh about 3 g more or less than their mean of 62 g.

(b) 62 − 53 = 9 g below, and $\frac{9}{3} = 3$: 3 standard deviations below the mean.

(c) Two standard deviations are 2 × 3 = 6 g, so the bakery sells croissants from 62 − 6 = 56 g to 62 + 6 = 68 g. A 67 g croissant is $\frac{5}{3} \approx 1.67$ standard deviations above the mean, inside the range, so it is sold.

SCORING

1 point: a sentence naming the croissants, the 3 g as a typical distance and the mean 62 g. 1 point: 3 standard deviations below. 1 point: 56 g to 68 g with the 67 g croissant sold.

6. A farm packs 2600 eggs a day. Isaac weighs 4 of them, chosen at random, to estimate the spread of the masses of all the day's eggs. Three of the masses are 55, 61, 63 grams. The fourth was smudged, but the mean of the four is 62 grams. Find the fourth mass, and the right standard deviation to two decimal places. (3 points)

ANSWER

The fourth mass is 69 grams; $s = \sqrt{\frac{100}{3}} \approx 5.77$ grams.

SOLUTION

The four masses add to 4 · 62 = 248 grams, and the three known ones add to 179, so the fourth is 248 − 179 = 69 grams.

The four eggs stand in for all 2600, so they are a sample with $n = 4$. The deviations from 62 are −7, −1, 1, 7, which add to 0. Their squares 49, 1, 1, 49 add to 100.

Divide by $n - 1 = 3$: $s^2 = \frac{100}{3}$ and $s = \sqrt{\frac{100}{3}} \approx 5.77$ grams.

SCORING

1 point: the fourth mass, 69 grams, from the mean. 1 point: the sum of squared deviations, 100. 1 point: treats the eggs as a sample, divides by 3 and gets $s \approx 5.77$ grams.

7. A school cafeteria has two lunch lines. A student timed the wait in each line, in minutes, on five days. Treat each line's five days as a population. (4 points)

	Mon	Tue	Wed	Thu	Fri
Hot line	3	5	4	7	6
Salad line	0	12	1	10	2

(a) Find the mean and standard deviation of each line's waits, to two decimal places.

(b) Compare the two lines in context.

(c) Elena must be through the line within 8 minutes to get to a club meeting. Which line should she choose? Use the standard deviations in your answer.

ANSWER

(a) Hot line: mean 5 min, standard deviation ≈ 1.41 min. Salad line: mean 5 min, standard deviation ≈ 4.98 min. (b) Both waits average 5 minutes; the salad line's wait is far less predictable. (c) The hot line.

SOLUTION

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(a) Hot line: the waits add to 25, so the mean is 5 min. The distances are 2, 0, 1, 2, 1, with squares adding to 4 + 0 + 1 + 4 + 1 = 10. The variance is $\frac{10}{5} = 2$, so the standard deviation is $\sqrt{2} \approx 1.41$ min.

Salad line: the waits add to 25, so the mean is 5 min. The distances are 5, 7, 4, 5, 3, with squares adding to 25 + 49 + 16 + 25 + 9 = 124. The variance is $\frac{124}{5} = 24.8$, so the standard deviation is $\sqrt{24.8} \approx 4.98$ min.

(b) Both lines have a mean wait of 5 minutes. The hot line is steady, its waits typically within about 1.41 minutes of that, while the salad line's waits typically stray about 4.98 minutes from it, more than three times as far.

(c) The hot line. Its usual wait, about 5 ± 1.41 minutes, stays well under 8 minutes, and it never took more than 7 minutes. The salad line's usual range, about 5 ± 4.98 minutes, reaches past 8, and its wait was over 8 minutes on two of the five days.

SCORING

1 point: both means and standard deviations correct. 1 point: work shown (squared distances or variance) for at least one line. 1 point: a comparison saying the mean waits are equal and the salad line is more spread, in minutes. 1 point: the hot line, with a reason that uses the standard deviations.

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