Statistics Unit 1 quiz 4: Shifting and rescaling a data set, z-scores and the empirical rule (answer key)

8 questions, 22 points

ANSWERS AT A GLANCE

1. (a) Standard deviation 12, variance 144. (b) He multiplied the variance by 3 instead of $3^2 = 9$, and he subtracted the 7, which has no effect on any spread.
2. (a) about 97.5% (b) about 15.85%, or about 16%
3. Mean 124 dollars, median 104 dollars, standard deviation 50 dollars, IQR 60 dollars.
4. Mean 70, median 75, standard deviation 10.
5. $b = 2$, $a = 10$; median rating 60; standard deviation 18.
6. (a) Above 900 kWh. (b) $z = 1.25$, so the home does not get a letter.
7. Long jump $z = 2.1$; sprint $z = -2$, which is 2 standard deviations faster than the mean. She did better in the long jump.
8. $\mu = 6{,}400$ steps, $\sigma = 1{,}600$ steps

WORKED SOLUTIONS

1. Every value in a data set with standard deviation 4 is multiplied by 3, and then 7 is subtracted. Kofi says the new variance is 3 × 16 − 7 = 41. (2 points)

(a) Find the correct new standard deviation and variance.

(b) Name both of Kofi's mistakes.

ANSWER

(a) Standard deviation 12, variance 144. (b) He multiplied the variance by 3 instead of $3^2 = 9$, and he subtracted the 7, which has no effect on any spread.

SOLUTION

(a) The rule is $y = 3x - 7$. The standard deviation feels only the multiplier: 3 × 4 = 12. The variance is $12^2 = 144$, which is also $3^2 \times 16$.

(b) Squaring the deviations squares the factor, so the variance is multiplied by 9, not 3. And taking away 7 from every value moves the data without changing any deviation, so the 7 does not belong in a variance at all.

SCORING

1 point for standard deviation 12 and variance 144; 1 point for naming both mistakes (factor 3 instead of 9, and the 7 in a spread).

2. The times that seeds of one pepper variety take to sprout in a greenhouse are modeled by $N(9, 1.5)$, in days. Use the empirical rule: for a normal model, about 68%, 95% and 99.7% of values lie within one, two and three standard deviations of the mean. (2 points)

(a) Estimate the share of seeds that take longer than 6 days to sprout.

(b) Estimate the share of seeds that sprout in between 4.5 and 7.5 days.

ANSWER

(a) about 97.5% (b) about 15.85%, or about 16%

SOLUTION

(a) 6 days is $z = \dfrac{6 - 9}{1.5} = -2$. The share below $z = -2$ is half of 100% − 95% = 5%, which is 2.5%, so about 97.5% take longer than 6 days.

(b) 4.5 days is $z = \dfrac{4.5 - 9}{1.5} = -3$ and 7.5 days is $z = \dfrac{7.5 - 9}{1.5} = -1$. From −3 to −2 is half of 99.7% − 95%, which is 2.35%, and from −2 to −1 is half of 95% − 68%, which is 13.5%. Together that is about 15.85%.

SCORING

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(a) 1 point for about 97.5%, from $z = -2$. (b) 1 point for about 15.85% (or about 16%), from $z = -3$ to $z = -1$.

3. School groups book seats at a play. Each ticket costs 10 dollars, and each group also pays one booking fee of 4 dollars. The group sizes had mean 12 students, median 10 students, standard deviation 5 students and IQR 6 students. Find the mean, median, standard deviation and IQR of the amounts, in dollars, that the groups paid. (3 points)

ANSWER
Mean 124 dollars, median 104 dollars, standard deviation 50 dollars, IQR 60 dollars.

SOLUTION
A group of x students pays $y = 4 + 10x$ dollars.

Centers go through the whole rule: mean 4 + 10 × 12 = 4 + 120 = 124 dollars, median 4 + 10 × 10 = 4 + 100 = 104 dollars.

Spreads feel only the multiplier: standard deviation 10 × 5 = 50 dollars, IQR 10 × 6 = 60 dollars. The booking fee cancels in every spread.

SCORING
1 point for the rule $y = 4 + 10x$; 1 point for mean 124 and median 104; 1 point for standard deviation 50 and IQR 60 (no booking fee added).

4. A teacher curved the scores on a hard test by the rule $y = 0.8x + 20$, where x is the original score. After the curve, the scores had mean 76, median 80 and standard deviation 8. Find the original mean, median and standard deviation. (3 points)

ANSWER
Mean 70, median 75, standard deviation 10.

SOLUTION
Undo the rule: take away 20, then divide by 0.8.

Mean: $76 = 0.8\bar{x} + 20$, so $\bar{x} = \dfrac{76 - 20}{0.8} = \dfrac{56}{0.8} = 70$. Median: $\dfrac{80 - 20}{0.8} = \dfrac{60}{0.8} = 75$.

The standard deviation only felt the 0.8, so only divide: $\dfrac{8}{0.8} = 10$.

SCORING
1 point for mean 70; 1 point for median 75; 1 point for standard deviation 10 (dividing by 0.8 without taking away 20).

5. A video game turns each player's raw points x into a rating $y = a + bx$, with $b > 0$, the same rule for every player. The raw points had $Q_1 = 20$, median 25, $Q_3 = 32$ and standard deviation 9. The ratings had $Q_1 = 50$ and $Q_3 = 74$. Find a and b, the median rating and the standard deviation of the ratings. (3 points)

ANSWER
$b = 2$, $a = 10$; median rating 60; standard deviation 18.

SOLUTION
The IQR sees only b: the raw IQR is 32 − 20 = 12 and the rating IQR is 74 − 50 = 24, so $24 = 12b$ and $b = 2$.

A quartile gives a: $50 = a + 2 \times 20$, so $a = 10$. Check with Q_3: 10 + 2 × 32 = 74.

Median rating 10 + 2 × 25 = 60; standard deviation 2 × 9 = 18.

SCORING

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1 point for $b = 2$ from the IQRs; 1 point for $a = 10$; 1 point for median 60 and standard deviation 18.

6. A power company writes to every home whose monthly electricity use has a z-score above 1.5, to offer a free energy check. Monthly use among its homes has mean 660 kWh (kilowatt-hours) and standard deviation 160 kWh. (3 points)

(a) Above what monthly use does a home get a letter?

(b) One home used 860 kWh last month. Find its z-score and say whether it gets a letter.

ANSWER

(a) Above 900 kWh. (b) $z = 1.25$, so the home does not get a letter.

SOLUTION

(a) The cut is the use whose z-score is 1.5: $x = \mu + z\sigma = 660 + (1.5)(160) = 900$ kWh. A home that uses more than 900 kWh gets a letter.

(b) $z = \dfrac{860 - 660}{160} = \dfrac{200}{160} = 1.25$. Compared with the cut of 1.5, the home does not get a letter.

SCORING

(a) 1 point for 900 kWh. (b) 1 point for $z = 1.25$; 1 point for the decision, which must match the student's z-score against 1.5.

7. At a county track meet, Sofia enters the long jump and the 100 m sprint. The long jumps at the meet have mean 5.20 m and standard deviation 0.30 m, and Sofia jumps 5.83 m. The sprint times have mean 13.4 seconds and standard deviation 0.5 seconds, and Sofia runs 12.4 seconds. In which event did she do better, compared with the other competitors? Show the z-scores. (3 points)

ANSWER

Long jump $z = 2.1$; sprint $z = -2$, which is 2 standard deviations faster than the mean. She did better in the long jump.

SOLUTION

Long jump: $z = \dfrac{5.83 - 5.20}{0.30} = \dfrac{0.63}{0.30} = 2.1$, so her jump is 2.1 standard deviations longer than the mean.

Sprint: $z = \dfrac{12.4 - 13.4}{0.5} = \dfrac{-1}{0.5} = -2$. In a sprint a lower time is better, so the negative sign is good news: she ran 2 standard deviations faster than the mean.

Both results are good. The long jump is 2.1 standard deviations to the good side and the sprint 2, so she did slightly better in the long jump.

SCORING

1 point for the long jump z-score 2.1. 1 point for the sprint z-score −2. 1 point for the conclusion (long jump), which must say that a negative sprint z-score is the good direction; comparing 2.1 with −2 as signed numbers without that reason earns 0 for this point.

8. The daily step counts that members of a walking club log on their fitness trackers have a distribution in which 7,200 steps has z-score 0.5 and 10,400 steps has z-score 2.5. Find the mean and the standard deviation of the daily step counts. (3 points)

ANSWER

$\mu = 6{,}400$ steps, $\sigma = 1{,}600$ steps

SOLUTION

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Through $x = \mu + z\sigma$, the two counts give

$$7{,}200 = \mu + 0.5\sigma,$$
$$10{,}400 = \mu + 2.5\sigma.$$

Taking the first from the second: $3{,}200 = (2.5 - 0.5)\sigma$, so $\sigma = 1{,}600$ steps. Then $\mu = 7{,}200 - 0.5(1{,}600) = 6{,}400$ steps.

SCORING

1 point for two correct equations $x = \mu + z\sigma$ (or the same reasoning in words: the counts are 2.5 – 0.5 standard deviations apart). 1 point for $\sigma = 1{,}600$. 1 point for $\mu = 6{,}400$.

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