Statistics Unit 1 quiz 6: Cumulative frequency graphs and percentiles (answer key)

4 questions, 11 points

ANSWERS AT A GLANCE

1. About 16.4 kg.
2. (a) About the 52.5th percentile (84 visitors). (b) About 8%, roughly 13 visitors.
3. (a) 12, 28, 42, 18 potatoes in 50 to 100, 100 to 150, 150 to 250, 250 to 400 g. (b) The 150 to 250 class holds the most; slopes 0.24, 0.56, 0.42, 0.12 potatoes per gram; the 100 to 150 segment is steepest. (c) Slope is count divided by width; the 150 to 250 class is twice as wide, so its 42 potatoes are spread more thinly. The steepest segment marks where potatoes are most crowded per gram.
4. (a) $Q_1 \approx 1.29$ hours, $Q_3 = 3$ hours, interquartile range about 1.71 hours. (b) Q_3: its target, 90 rentals, is exactly the running total at the boundary 3 hours, so it is read at a point the table gives exactly.

WORKED SOLUTIONS

1. An airline weighed the 65 suitcases checked on one flight, in kilograms. A mass on a class boundary goes in the class to its right.

Mass (kg)	6 to 10	10 to 14	14 to 18	18 to 22	22 to 26
Suitcases	5	21	22	14	3

Use the cumulative frequency graph of these counts to estimate the 60th percentile of the masses, to the nearest tenth of a kilogram. (2 points)

ANSWER

About 16.4 kg.

SOLUTION

The target count is $\frac{60}{100} \cdot 65 = 39$ suitcases. The running totals are 5 at 10 kg, 26 at 14 kg and 48 at 18 kg, so the target lies in the 14 to 18 class, 39 − 26 = 13 suitcases into its 22:

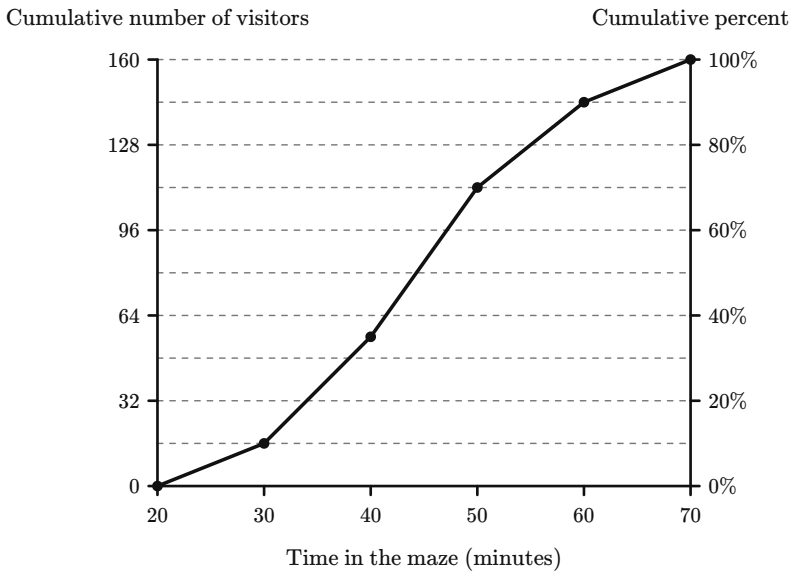
$$P_{60} \approx 14 + \frac{13}{22} \cdot 4 \approx 16.4 \text{ kg}.$$

SCORING

1 point for the target count 39 placed in the 14 to 18 class; 1 point for about 16.4 kg.

2. The cumulative frequency graph shows how long each of 160 visitors took to get through a corn maze. Each plotted point lies on a faint line or halfway between two. (3 points)

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(a) Estimate the percentile rank of a 45 minute visit.

(b) Estimate the percent of visitors who took longer than 62 minutes, and about how many visitors that is.

ANSWER

(a) About the 52.5th percentile (84 visitors). (b) About 8%, roughly 13 visitors.

SOLUTION

The graph passes through (20, 0), (30, 16), (40, 56), (50, 112), (60, 144) and (70, 160).

(a) 45 minutes is halfway through the 40 to 50 class, which climbs from 56 to 112, a class of 56 visitors:

$$56 + \frac{5}{10} \cdot 56 = 84 \text{ visitors}, \qquad \frac{84}{160} = 52.5\%.$$

(b) 62 minutes is $\frac{2}{10}$ of the way through the 60 to 70 class, which holds 160 − 144 = 16 visitors:

$$144 + \frac{2}{10} \cdot 16 = 147.2 \text{ visitors}, \qquad \frac{147.2}{160} = 92\%.$$

So about 100 − 92 = 8% of visitors took longer than 62 minutes, about 160 − 147.2 = 12.8, roughly 13 visitors.

SCORING

1 point for 84 visitors and the 52.5th percentile in (a); 1 point for about 147.2 visitors below 62 minutes; 1 point for about 8% and about 13 visitors longer.

3. A farm weighed 100 potatoes from one harvest. Its cumulative frequency graph, with classes of different widths, passes through the points below. (3 points)

Mass (g)	**50**	**100**	**150**	**250**	**400**
Potatoes lighter than this	0	12	40	82	100

(a) Find the number of potatoes in each class.

(b) Which class holds the most potatoes? Find the slope of each segment of the graph, and say which segment is steepest.

(c) Explain why the steepest segment is not the class with the most potatoes, and what the steepest segment does show.

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ANSWER

(a) 12, 28, 42, 18 potatoes in 50 to 100, 100 to 150, 150 to 250, 250 to 400 g. (b) The 150 to 250 class holds the most; slopes 0.24, 0.56, 0.42, 0.12 potatoes per gram; the 100 to 150 segment is steepest. (c) Slope is count divided by width; the 150 to 250 class is twice as wide, so its 42 potatoes are spread more thinly. The steepest segment marks where potatoes are most crowded per gram.

SOLUTION

(a) Each class count is the climb across it: 12 − 0 = 12, 40 − 12 = 28, 82 − 40 = 42 and 100 − 82 = 18 potatoes. They add to 100.

(b) The 150 to 250 g class holds the most, 42. A class with f values and width w climbs with slope $\dfrac{f}{w}$:

$$\frac{12}{50} = 0.24, \quad \frac{28}{50} = 0.56, \quad \frac{42}{100} = 0.42, \quad \frac{18}{150} = 0.12$$

potatoes per gram. The 100 to 150 g segment is steepest.

(c) The slope is the count per gram of class width, not the count. The 150 to 250 g class has more potatoes, 42 against 28, but it is twice as wide, so each 50 g of it holds only 21 on average. The steepest segment shows where the masses are most crowded, here 100 to 150 g.

SCORING

1 point for the four counts; 1 point for naming the 150 to 250 class and the 100 to 150 segment with correct slopes; 1 point for explaining that slope is count per unit of width, so a wider class climbs less steeply.

4. A lake's canoe hire logged its 120 rentals last month, in hours. A time on a class boundary goes in the class to its right. (3 points)

Rental length (hours)	0 to 1	1 to 2	2 to 3	3 to 4	4 to 6
Rentals	18	42	30	18	12

(a) Use the cumulative frequency graph to estimate Q_1, Q_3 and the interquartile range, to the nearest hundredth of an hour.

(b) One of your two quartile estimates does not rest on the evenly spread assumption. Which one, and why?

ANSWER

(a) $Q_1 \approx 1.29$ hours, $Q_3 = 3$ hours, interquartile range about 1.71 hours. (b) Q_3: its target, 90 rentals, is exactly the running total at the boundary 3 hours, so it is read at a point the table gives exactly.

SOLUTION

(a) The running totals are 18, 60, 90, 108 and 120 at 1, 2, 3, 4 and 6 hours. Q_1 has target $0.25 \cdot 120 = 30$, between 18 and 60: 12 rentals into the 42 of the 1 to 2 class. Q_3 has target $0.75 \cdot 120 = 90$, which is the running total at 3 hours:

$$Q_1 \approx 1 + \frac{12}{42} \cdot 1 \approx 1.29 \text{ hours},$$
$$Q_3 = 3 \text{ hours}.$$

The interquartile range is about 3 − 1.2857 ≈ 1.71 hours.

(b) Q_3. The graph's height at a boundary is a count straight from the table: exactly 90 rentals, 75%, lasted less than 3 hours, however the times sit inside the classes. Reading Q_3 needs no splitting of a class. Q_1 places 12 of the 42 rentals of 1 to 2 hours by assuming they are spread evenly through the class.

SCORING

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> 1 point for $Q_1 \approx 1.29$ hours; 1 point for $Q_3 = 3$ hours and the interquartile range about 1.71 hours; 1 point for naming Q_3 because 90 is a running total at a boundary.

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