Statistics Unit 1 worksheet: Computing a standard deviation, sample and population (answer key)

ANSWERS AT A GLANCE

1. $\sigma = \sqrt{\frac{84}{5}} \approx 4.10$ years
2. $s = \sqrt{14.25} \approx 3.77$ kg
3. (a) He divided by one less than the number of rows ($4 - 1 = 3$) instead of one less than the number of students. (b) $s = \sqrt{\frac{20}{24}} \approx 0.91$ pets
4. (a) 15 crossings (b) $4\sqrt{\frac{14}{15}} \approx 3.86$ minutes
5. (a) $\sigma = \sqrt{9.2} \approx 3.03$ bowls (b) $\sigma = \sqrt{\frac{46}{6}} \approx 2.77$ bowls (c) The new value equals the mean, so it adds 0 to the sum of squared deviations, but the sum is now divided by 6 instead of 5.
6. (a) Both $\sigma = \sqrt{\frac{26}{3}} \approx 2.94$ (b) A: $s = \sqrt{13} \approx 3.61$; B: $s = \sqrt{10.4} \approx 3.22$ (c) Doubling doubles both the sum of squared deviations and the count, so their ratio stays the same; but $n - 1$ goes from 2 to 5, not from 2 to 4.
7. 7, 7, 13, 13

WORKED SOLUTIONS

1. The five teachers in a small school's music department have taught for these numbers of years: 6, 8, 11, 12, 18. The five are the whole department, the group being described. Find the standard deviation of their years of teaching, to two decimal places.

ANSWER

$\sigma = \sqrt{\frac{84}{5}} \approx 4.10$ years

SOLUTION

The five teachers are the whole group being described, so they are a population and we divide by $N = 5$. The mean is $\mu = \dfrac{55}{5} = 11$ years.

The deviations from 11 are −5, −3, 0, 1, 7, which add to 0, a check on the mean. Their squares 25, 9, 0, 1, 49 add to 84.

So $\sigma^2 = \dfrac{84}{5}$ and $\sigma = \sqrt{\frac{84}{5}} \approx 4.10$ years. It is sensible: the largest distance from the mean is 7 years, and σ is below it.

2. A beekeeper has 60 hives. To estimate the spread of the honey harvest across all of them, she weighs the honey from 4 hives chosen at random: 9, 12, 14, 18 kg. Find the standard deviation of the harvest, to two decimal places.

ANSWER

$s = \sqrt{14.25} \approx 3.77$ kg

SOLUTION

The four hives stand in for all 60, so they are a sample with $n = 4$. The mean is $\bar{x} = \dfrac{53}{4} = 13.25$ kg, kept exact.

kg x	deviation $x - 13.25$	squared deviation
9	−4.25	18.0625
12	−1.25	1.5625
14	0.75	0.5625
18	4.75	22.5625
sum	0	42.75

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Divide by $n-1=3$: $s^2=\dfrac{42.75}{3}=14.25$, so $s=\sqrt{14.25}\approx 3.77$ kg.

3. A school has 600 students. Kevin asked a random sample of 25 of them how many pets they have.

pets x	**0**	**1**	**2**	**3**
students f	4	10	8	3

He found $\bar{x}=1.4$ and a column of fd^2 that adds to 20, both correct. Then he wrote $s=\sqrt{\frac{20}{3}}\approx 2.58$ pets.

(a) What is Kevin's mistake?

(b) Find the correct standard deviation, to two decimal places.

ANSWER

(a) He divided by one less than the number of rows (4 − 1 = 3) instead of one less than the number of students. (b) $s=\sqrt{\frac{20}{24}}\approx 0.91$ pets

SOLUTION

Check his numbers: the total of pets is $0\cdot 4+1\cdot 10+2\cdot 8+3\cdot 3=35$, so $\bar{x}=\dfrac{35}{25}=1.4$. The deviations are −1.4, −0.4, 0.6, 1.6, their squares 1.96, 0.16, 0.36, 2.56, and times the counts 7.84, 1.6, 2.88, 7.68, which add to 20.

(a) The table has 4 rows, and Kevin divided by 4 − 1 = 3. Each row stands for several students; the sample size is the total of the counts, $n=25$, so the divisor is $n-1=24$.

(b) $s^2=\dfrac{20}{24}\approx 0.833$ and $s=\sqrt{\frac{20}{24}}\approx 0.91$ pets. Kevin's 2.58 was also far too big to be sensible: no student is more than 1.6 pets from the mean.

4. A ferry company times a random sample of its crossings, in minutes. The squared deviations from the sample mean add to 224, and the sample standard deviation is exactly $s=4$ minutes.

(a) How many crossings are in the sample?

(b) What would dividing by the number of crossings, instead of one less, have given? Give it to two decimal places.

ANSWER

(a) 15 crossings (b) $4\sqrt{\frac{14}{15}}\approx 3.86$ minutes

SOLUTION

(a) For a sample, $s^2=\dfrac{\text{sum of squared deviations}}{n-1}$. Here $s^2=4^2=16$, so

$$16=\frac{224}{n-1},\qquad n-1=\frac{224}{16}=14,\qquad n=15$$

There are 15 crossings in the sample.

(b) Dividing the same sum by 15 instead of 14 multiplies the squared figure by $\dfrac{14}{15}$, so the result is $\sqrt{16\cdot\frac{14}{15}}=4\sqrt{\frac{14}{15}}\approx 3.86$ minutes. It is a little smaller than $s=4$, because the same sum is divided by a bigger number.

5. The five students in a pottery class made 4, 8, 9, 11 and 13 bowls this term. Treat the class as a population.

(a) Find the standard deviation of the number of bowls, to two decimal places.

(b) A sixth student joins the class and has made 9 bowls. Find the standard deviation for the six students.

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(c) No student's count changed, yet the standard deviation did. Explain why it went the way it did.

ANSWER

(a) $\sigma = \sqrt{9.2} \approx 3.03$ bowls (b) $\sigma = \sqrt{\frac{46}{6}} \approx 2.77$ bowls (c) The new value equals the mean, so it adds 0 to the sum of squared deviations, but the sum is now divided by 6 instead of 5.

SOLUTION

(a) The mean is $\mu = \frac{45}{5} = 9$ bowls. The deviations −5, −1, 0, 2, 4 add to 0, and their squares 25, 1, 0, 4, 16 add to 46. So $\sigma^2 = \frac{46}{5} = 9.2$ and $\sigma = \sqrt{9.2} \approx 3.03$ bowls.

(b) The six counts add to 54, so the mean is still $\frac{54}{6} = 9$. The new student's deviation is 0, so the squared deviations still add to 46. Now $\sigma^2 = \frac{46}{6} \approx 7.67$ and $\sigma = \sqrt{\frac{46}{6}} \approx 2.77$ bowls.

(c) A value at the mean leaves the mean and the sum of squared deviations unchanged, but it raises the count. The same total spread is shared out over more values, so the typical distance from the mean, and with it σ, goes down.

6. Data set A is 3, 5, 10. Data set B lists each of those numbers twice: 3, 3, 5, 5, 10, 10.

(a) Treat each set as a population and find σ for each, to two decimal places.

(b) Treat each set as a sample and find s for each, to two decimal places.

(c) Explain why doubling the data left σ alone but changed s.

ANSWER

(a) Both $\sigma = \sqrt{\frac{26}{3}} \approx 2.94$ (b) A: $s = \sqrt{13} \approx 3.61$; B: $s = \sqrt{10.4} \approx 3.22$ (c) Doubling doubles both the sum of squared deviations and the count, so their ratio stays the same; but $n - 1$ goes from 2 to 5, not from 2 to 4.

SOLUTION

Both sets have mean 6. For A the deviations −3, −1, 4 have squares 9, 1, 16, adding to 26. B has each of these twice, so its squared deviations add to 52.

(a) A: $\sigma = \sqrt{\frac{26}{3}} \approx 2.94$. B: $\sigma = \sqrt{\frac{52}{6}} = \sqrt{\frac{26}{3}} \approx 2.94$, the same.

(b) A: $s = \sqrt{\frac{26}{2}} = \sqrt{13} \approx 3.61$. B: $s = \sqrt{\frac{52}{5}} = \sqrt{10.4} \approx 3.22$.

(c) Doubling the data doubles the sum of squared deviations and doubles the count, so the population divisor doubles too and $\frac{52}{6} = \frac{26}{3}$. The sample divisor is one less than the count: it goes from 2 to 5, which is more than double, so s drops. The $n - 1$ makes a bigger difference to a small sample than to a large one.

7. A population of four whole numbers has mean 10 and standard deviation $\sigma = 3$. None of the four numbers is more than 3 away from the mean. Find the four numbers, and explain why no other four numbers work.

ANSWER

7, 7, 13, 13

SOLUTION

Since $\sigma^2 = 9$ and $N = 4$, the squared deviations add to $4 \cdot 9 = 36$. No number is more than 3 from the mean, so each squared deviation is at most 9. Four numbers each at most 9 can add to 36 only if every one is exactly 9, so every deviation is 3 or −3, and every number is 7 or 13.

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The deviations must add to 0, so there are as many −3s as 3s: two of each. The numbers are 7, 7, 13, 13. Check: the mean is $\frac{40}{4} = 10$ and $\sigma = \sqrt{\frac{36}{4}} = 3$. Here σ equals the largest distance from the mean, which happens only when every value is that same distance away.

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