Statistics Unit 1 worksheet: Interpreting a standard deviation (answer key)

ANSWERS AT A GLANCE

1. (a) 2 above (b) 1.5 below (c) 2.5 above (d) the 31-second shot (e) 22 seconds
2. (a) Mean 22 g, standard deviation 2.5 g. (b) The 8 letters typically weigh about 2.5 g more or less than their mean of 22 g. (c) 6 letters. (d) MAD 2.25 g; the standard deviation squares the distances, so the larger ones count for more.
3. (a) Diego treats the standard deviation as a limit, but it is a typical distance, and some apples sit further out. Priya treats the spread as the weight; a typical apple weighs about 180 g. Amara uses a spread to compare centers; the standard deviation says nothing about which farm's apples are heavier. (b) The orchard's apples typically weigh about 15 g more or less than their mean of 180 g.
4. (a) 5, 5, 5, 5, 5. (b) 0, 0, 5, 10, 10, standard deviation $\sqrt{20} \approx 4.47$. (c) Any five whole-number scores from 0 to 10 that add to 25 and whose squared distances from 5 add to 20, for example 2, 4, 5, 6, 8 or 1, 6, 6, 6, 6.
5. (a) Mean 15 min, standard deviation ≈ 16.0 min. (b) Mean 9 min, standard deviation 2 min. (c) With (a): about 0.06 standard deviations below the mean. With (b): 2.5 standard deviations above. (d) The summary from (b): the one undocked ride inflates the mean and the standard deviation so much that a long ride looks ordinary.
6. (a) No score sits more than 1 from the mean, so the standard deviation is at most 1. (b) About 0.89, from 7, 7, 8, 9, 9.
7. (a) Isaac: mean 64 s, standard deviation ≈ 1.29 s. Omar: mean 62 s, standard deviation ≈ 2.65 s. (b) Omar is faster on average, by 2 s. Isaac is more consistent: his times typically sit about 1.29 s from his mean, Omar's about 2.65 s from his, roughly twice as far. (c) Isaac about 62.7 to 65.3 s, Omar about 59.4 to 64.6 s; Isaac's.

WORKED SOLUTIONS

1. A café's espresso machine pulls shots that last a mean of 26 seconds, with a standard deviation of 2 seconds. For each shot time in (a) to (c), say how many standard deviations it sits from the mean, and on which side.

(a) 30 seconds (b) 23 seconds (c) 31 seconds

(d) Which of the three shots is the most unusual for this machine?

(e) What shot time is 2 standard deviations below the mean?

ANSWER

(a) 2 above (b) 1.5 below (c) 2.5 above (d) the 31-second shot (e) 22 seconds

SOLUTION

Divide each distance from the mean by the standard deviation, 2 seconds.

(a) 30 − 26 = 4 seconds above, and $\frac{4}{2} = 2$, so the count is 2, above the mean.

(b) 26 − 23 = 3 seconds below, and $\frac{3}{2} = 1.5$: 1.5 standard deviations below the mean.

(c) 31 − 26 = 5 seconds above, and $\frac{5}{2} = 2.5$: 2.5 standard deviations above the mean.

(d) The 31-second shot, at 2.5 standard deviations, is the furthest from usual.

(e) 26 − 2 × 2 = 22 seconds.

2. A post office weighed 8 letters, in grams: 18, 20, 20, 21, 23, 24, 24, 26. Treat the 8 letters as a population.

(a) Find the mean and the standard deviation of the weights.

(b) Write one sentence interpreting the standard deviation.

(c) How many of the letters weigh within one standard deviation of the mean?

(d) Find the mean absolute deviation (MAD) of the weights. Explain why it comes out smaller than the standard deviation.

ANSWER

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(a) Mean 22 g, standard deviation 2.5 g. (b) The 8 letters typically weigh about 2.5 g more or less than their mean of 22 g. (c) 6 letters. (d) MAD 2.25 g; the standard deviation squares the distances, so the larger ones count for more.

SOLUTION

(a) The weights add to 176, so the mean is $\frac{176}{8} = 22$ g. The distances from 22 are 4, 2, 2, 1, 1, 2, 2, 4, and their squares 16, 4, 4, 1, 1, 4, 4, 16 add to 50. The variance is $\frac{50}{8} = 6.25$ g^2, so the standard deviation is $\sqrt{6.25} = 2.5$ g.

(b) The 8 letters typically weigh about 2.5 g more or less than their mean of 22 g.

(c) Within one standard deviation means from 22 − 2.5 = 19.5 g to 22 + 2.5 = 24.5 g. The letters of 20, 20, 21, 23, 24, 24 g are inside; 18 g and 26 g are outside. So 6 of the 8 letters.

(d) The distances add to 18, so the MAD is $\frac{18}{8} = 2.25$ g. The standard deviation squares the distances before averaging, which gives the two 4 g distances more weight than the 1 g ones; the MAD treats every distance alike. The two would be equal only if every letter sat the same distance from the mean.

3. An orchard reports that its apples this season have a mean weight of 180 g and a standard deviation of 15 g. Three students read the report.

Diego: "Every apple weighs between 165 g and 195 g."

Priya: "A typical apple weighs 15 g."

Amara: "The farm next door reports a standard deviation of 10 g, so our apples are heavier than theirs."

(a) Explain what each student has mixed up.

(b) Write one correct sentence about what the 15 g says.

ANSWER

(a) Diego treats the standard deviation as a limit, but it is a typical distance, and some apples sit further out. Priya treats the spread as the weight; a typical apple weighs about 180 g. Amara uses a spread to compare centers; the standard deviation says nothing about which farm's apples are heavier. (b) The orchard's apples typically weigh about 15 g more or less than their mean of 180 g.

SOLUTION

(a) Diego: 180 − 15 = 165 and 180 + 15 = 195, but the standard deviation is a rough typical distance from the mean, not the furthest any apple can sit. It blends all the distances, so some apples sit more than 15 g from 180 g.

Priya: the 15 g measures how far weights stray from the mean, not the weight itself. A typical apple weighs about 180 g, give or take about 15 g.

Amara: a standard deviation measures spread only. The smaller 10 g says the neighbor's apples are more alike in weight, but which farm's apples are heavier is a question for the two means, and the neighbor's mean is not given.

(b) The orchard's apples this season typically weigh about 15 g more or less than their mean of 180 g.

4. Five judges each give a skater a whole-number score from 0 to 10. The mean of the five scores is 5.

(a) Give five scores with a standard deviation of 0.

(b) Give five scores with the largest standard deviation possible, find it to two decimal places, and explain why no set of scores does better.

(c) Give five scores with a standard deviation of exactly 2.

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ANSWER

(a) 5, 5, 5, 5, 5. (b) 0, 0, 5, 10, 10, standard deviation $\sqrt{20} \approx 4.47$. (c) Any five whole-number scores from 0 to 10 that add to 25 and whose squared distances from 5 add to 20, for example 2, 4, 5, 6, 8 or 1, 6, 6, 6, 6.

SOLUTION

(a) A standard deviation of 0 means no score strays from the mean: 5, 5, 5, 5, 5.

(b) No score can sit more than 5 from the mean, since the scores run from 0 to 10, so each squared distance is at most 25. The distances above the mean must balance those below it. All five scores cannot be 0 or 10: five distances of ± 5 never add to 0. With four scores at 0 or 10, two must be 0 and two 10, and the fifth is 5. The squared distances are 25, 25, 0, 25, 25, adding to 100. With three scores at the ends, their distances add to ± 5 (or ± 15, which the other two cannot balance), so the other two, each at most 4 away, must add to ∓ 5: at best 4 and 1, giving at most 75 + 16 + 1 = 92. With two or fewer at the ends the total is at most 50 + 3 × 16 = 98. So 100 is the most, the variance is $\frac{100}{5} = 20$ and the standard deviation $\sqrt{20} \approx 4.47$.

(c) A standard deviation of 2 means a variance of 4, so the squared distances from 5 must add to $5 \times 2^2 = 20$, while the scores still add to 25 to keep the mean 5. Distances 3, 1, 0, 1, 3 give 9 + 1 + 0 + 1 + 9 = 20, so 2, 4, 5, 6, 8 works. Distances 4, 1, 1, 1, 1 (one below, four above) give 16 + 4 = 20, so 1, 6, 6, 6, 6 works too. Any set meeting both conditions is correct.

5. A bike-share station logged eight rides, in minutes: 9, 6, 11, 57, 8, 12, 7, 10. The 57-minute ride was a rider who forgot to dock the bike. Treat the rides as a population.

(a) Find the mean and standard deviation of all eight rides, the standard deviation to one decimal place.

(b) Find the mean and standard deviation of the seven other rides.

(c) A new ride lasts 14 minutes. Count how many standard deviations it sits from the mean using the summary from (a), and again using the summary from (b).

(d) Which summary shows honestly how unusual a 14-minute ride is? Explain.

ANSWER

(a) Mean 15 min, standard deviation ≈ 16.0 min. (b) Mean 9 min, standard deviation 2 min. (c) With (a): about 0.06 standard deviations below the mean. With (b): 2.5 standard deviations above. (d) The summary from (b): the one undocked ride inflates the mean and the standard deviation so much that a long ride looks ordinary.

SOLUTION

(a) The rides add to 120, so the mean is $\frac{120}{8} = 15$ min. The distances from 15 are 6, 9, 4, 42, 7, 3, 8, 5, with squares 36, 81, 16, 1764, 49, 9, 64, 25 adding to 2044. The variance is $\frac{2044}{8} = 255.5$, so the standard deviation is $\sqrt{255.5} \approx 16.0$ min.

(b) The seven rides add to 63, so the mean is 9 min. The distances are 0, 3, 2, 1, 3, 2, 1, with squares adding to 0 + 9 + 4 + 1 + 9 + 4 + 1 = 28. The variance is $\frac{28}{7} = 4$, so the standard deviation is 2 min.

(c) With (a): $\frac{14 - 15}{16.0} \approx -0.06$, so about 0.06 standard deviations below the mean. With (b): $\frac{14 - 9}{2} = 2.5$ standard deviations above the mean.

(d) The summary from (b). Every real ride lasted from 6 to 12 minutes, so 14 minutes is longer than any of them. The single 57-minute ride adds 1764 of the 2044 in (a), pulling the mean up to 15 and the standard

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deviation up to 16, so the summary from (a) calls a 14-minute ride slightly below average. A typical ride does not sit 16 minutes from 15 minutes; that description fits no ride at all.

6. Luis took five quizzes. Every score was a whole number from 7 to 9, and the mean was 8. He reports that the standard deviation of his scores is 1.2.

(a) Explain why the 1.2 must be a mistake.

(b) Find the largest standard deviation his five scores could have, to two decimal places, and give scores that reach it.

ANSWER

(a) No score sits more than 1 from the mean, so the standard deviation is at most 1. (b) About 0.89, from 7, 7, 8, 9, 9.

SOLUTION

(a) Every score is 7, 8 or 9, so every distance from the mean 8 is at most 1, and every squared distance at most 1. Their mean, the variance, is then at most 1, and so is its square root. The standard deviation blends the distances and can never be larger than the largest of them, so 1.2 is impossible.

(b) Each distance is 0 or 1. To keep the mean at 8, scores of 9 must be balanced by as many scores of 7, so with five scores at most two are 9 and two are 7, and one must be 8. The squared distances are 1, 1, 0, 1, 1, adding to 4, so the variance is $\frac{4}{5} = 0.8$ and the standard deviation is $\sqrt{0.8} \approx 0.89$. Scores: 7, 7, 8, 9, 9.

7. Two runners timed their last six 400 m races, in seconds. Treat each runner's six races as a population.

Isaac	**62**	**63**	**64**	**64**	**65**	**66**
Omar	58	60	61	63	64	66

(a) Find the mean and standard deviation of each runner's times, to two decimal places.

(b) Compare the two runners in two sentences, one about the centers and one about the spreads.

(c) Using one standard deviation on each side of the mean, give a rough range for each runner's next time. Whose next time can be predicted more closely?

ANSWER

(a) Isaac: mean 64 s, standard deviation ≈ 1.29 s. Omar: mean 62 s, standard deviation ≈ 2.65 s. (b) Omar is faster on average, by 2 s. Isaac is more consistent: his times typically sit about 1.29 s from his mean, Omar's about 2.65 s from his, roughly twice as far. (c) Isaac about 62.7 to 65.3 s, Omar about 59.4 to 64.6 s; Isaac's.

SOLUTION

(a) Isaac: the times add to 384, so the mean is $\frac{384}{6} = 64$ s. The distances are 2, 1, 0, 0, 1, 2, with squares adding to 4 + 1 + 0 + 0 + 1 + 4 = 10. The variance is $\frac{10}{6} \approx 1.667$, so the standard deviation is $\sqrt{1.667} \approx 1.29$ s.

Omar: the times add to 372, so the mean is $\frac{372}{6} = 62$ s. The distances are 4, 2, 1, 1, 2, 4, with squares adding to 16 + 4 + 1 + 1 + 4 + 16 = 42. The variance is $\frac{42}{6} = 7$, so the standard deviation is $\sqrt{7} \approx 2.65$ s.

(b) Omar is the faster runner on average: his times center on 62 s against Isaac's 64 s. Isaac is the more consistent: his times typically sit about 1.29 s from his mean, while Omar's typically sit about 2.65 s from his, roughly twice as far.

(c) Isaac: 64 ± 1.29, about 62.7 to 65.3 s. Omar: 62 ± 2.65, about 59.4 to 64.6 s. Isaac's range is about half as wide, so his next time can be predicted more closely, even though Omar will probably run faster. These

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> ranges are rough: a time can fall outside them.

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