Statistics Unit 1 worksheet: Making a histogram and choosing the bin width (answer key)

ANSWERS AT A GLANCE

1. (a) Categorical: bar graph. (b) Numerical: histogram. (c) Categorical: bar graph. (d) Numerical: histogram.
2. (a) First edge 5; 8 bins; the last bin is 40 to 45.
   (b) 5 to 10: 3; 10 to 15: 3; 15 to 20: 4; 20 to 25: 3; 25 to 30: 2; 30 to 35: 2; 35 to 40: 0; 40 to 45: 1.
   (c) Horizontal axis "Time on one foot (s)", edges 5, 10, ..., 45 at equal spacing; vertical axis "Number of students" from 0 to at least 4. Touching bars of heights 3, 3, 4, 3, 2, 2, 0, 1; the bin 35 to 40 is a gap at height zero.
3. (a) She counted the two 20 cm spans in "18 to 20", but 20 is an edge and belongs in "20 to 22"; and she left out the empty bin "22 to 24".
   (b) 14 to 16: 2; 16 to 18: 3; 18 to 20: 4; 20 to 22: 3; 22 to 24: 0; 24 to 26: 2.
   (c) Moving a value from one bin to another leaves the total the same, and a missing empty bin would only have added 0.
4. (a) 240 to 260 g.
   (b) The lightest is in the first bin: at least 120 g and less than 140 g, so 120 to 139 g. The heaviest is in the last bin: 240 to 259 g.
   (c) The biggest possible difference is 259 − 120 = 139 g, less than 145 g.
5. (a) One possible set: 130, 137, 140, 140, 144, 147, 149, 160, 163, 168. Any set works with 2 values from 130 to 139, 5 from 140 to 149, none from 150 to 159, 3 from 160 to 169, and at least three of them equal to 130, 140 or 160.
   (b) 150 would go in "150 to 160", which must be empty; 170 would go in "170 to 180", a fifth bin the histogram does not have.

WORKED SOLUTIONS

1. Priya helped at a school bike safety check and recorded four things about each of the 20 bikes checked:

(a) the brand (four brands); (b) the seat height, in centimeters; (c) the color of its frame; (d) its mass, in kilograms.

For each, say whether the data are numerical or categorical, and whether she should show them in a histogram or a bar graph.

> **ANSWER**
> (a) Categorical: bar graph. (b) Numerical: histogram. (c) Categorical: bar graph. (d) Numerical: histogram.
>
> **SOLUTION**
> A histogram's bins are stretches of a number line, so it needs numbers that can be placed on one.
>
> (a) The brands are names, not numbers: categorical, so a bar graph, one bar per brand.
>
> (b) A height in centimeters is a measurement: numerical, so a histogram, with bins such as 70 to 80 cm.
>
> (c) Colors are names: categorical, so a bar graph.
>
> (d) A mass in kilograms is a measurement: numerical, so a histogram.

2. Jasmine timed how long each of 18 classmates could stand on one foot with their eyes closed, in whole seconds:

7, 23, 14, 31, 9, 18, 25, 12, 40, 16, 21, 5, 28, 15, 19, 34, 11, 20

Use bins 5 seconds wide, with a value on an edge going in the bar to its right.

(a) Give the first edge, the number of bins and the last bin. (b) Make the frequency table. (c) Describe the histogram: name both axes, give the height of every bar in order, and say which bin shows as a gap.

> **ANSWER**
> (a) First edge 5; 8 bins; the last bin is 40 to 45.
>
> (b) 5 to 10: 3; 10 to 15: 3; 15 to 20: 4; 20 to 25: 3; 25 to 30: 2; 30 to 35: 2; 35 to 40: 0; 40 to 45: 1.

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(c) Horizontal axis "Time on one foot (s)", edges 5, 10, ..., 45 at equal spacing; vertical axis "Number of students" from 0 to at least 4. Touching bars of heights 3, 3, 4, 3, 2, 2, 0, 1; the bin 35 to 40 is a gap at height zero.

SOLUTION

(a) The smallest value is 5, itself a multiple of 5, so the first edge is 5. The largest, 40, sits on an edge, so it belongs to "40 to 45" and that bin is needed. The edges run 5, 10, ..., 45, so there are $\frac{45-5}{5} = 8$ bins.

(b) Sorted: 5, 7, 9, 11, 12, 14, 15, 16, 18, 19, 20, 21, 23, 25, 28, 31, 34, 40.

Time (s)	Values	Count
5 to 10	5, 7, 9	3
10 to 15	11, 12, 14	3
15 to 20	15, 16, 18, 19	4
20 to 25	20, 21, 23	3
25 to 30	25, 28	2
30 to 35	31, 34	2
35 to 40	none	0
40 to 45	40	1

The values 5, 15, 20, 25 and 40 sit on edges and each starts its bin. Check: 3 + 3 + 4 + 3 + 2 + 2 + 0 + 1 = 18.

(c) The horizontal axis is a number line from 5 to 45 with the nine edges equally spaced, named "Time on one foot (s)". The largest count is 4, so the vertical axis, "Number of students", runs from 0 to 4 (or 5) in steps of 1. The bars touch and have heights 3, 3, 4, 3, 2, 2, 0, 1. The bin 35 to 40 keeps its place as a gap, so the 40-second classmate stands apart.

3. Sarah measured the hand spans of 14 students, in whole centimeters:

18, 15, 21, 16, 20, 25, 18, 14, 19, 17, 24, 20, 16, 18

She made this frequency table with bins 2 cm wide:

Hand span (cm)	Count
14 to 16	2
16 to 18	3
18 to 20	6
20 to 22	1
24 to 26	2

Her counts add to 14, the number of students, but the table has two mistakes.

(a) Find both mistakes. (b) Write the correct table, with a value on an edge going in the bar to its right. (c) Explain why checking the total did not catch either mistake.

ANSWER

(a) She counted the two 20 cm spans in "18 to 20", but 20 is an edge and belongs in "20 to 22"; and she left out the empty bin "22 to 24".

(b) 14 to 16: 2; 16 to 18: 3; 18 to 20: 4; 20 to 22: 3; 22 to 24: 0; 24 to 26: 2.

(c) Moving a value from one bin to another leaves the total the same, and a missing empty bin would only have added 0.

SOLUTION

Sorted: 14, 15, 16, 16, 17, 18, 18, 18, 19, 20, 20, 21, 24, 25.

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(a) "18 to 20" holds 18, 18, 18, 19, which is 4 values, not 6: the extra two are the 20s, which sit on the edge 20 and go in the bar to its right. Also the table jumps from "20 to 22" to "24 to 26": no span lies from 22 up to 24, but that bin must still be listed with a count of 0.

(b)

Hand span (cm)	Count
14 to 16	2
16 to 18	3
18 to 20	4
20 to 22	3
22 to 24	0
24 to 26	2

Check: 2 + 3 + 4 + 3 + 0 + 2 = 14.

(c) The total only shows that each value was counted once. Putting the 20s in the wrong bin still counts each once, so the total stays 14; and the forgotten bin has a count of 0, so leaving it out changes nothing in the sum. The bins themselves have to be checked against the edge rule and for gaps.

4. Ethan weighed the 22 apples in a crate, each to the nearest gram, and drew a histogram the lesson's way: the first edge is the largest multiple of the width at or below the lightest apple, bins are added until one holds the heaviest, and a value on an edge goes in the bar to its right. His histogram has 7 bins, each 20 g wide, and its first edge is 120 g.

(a) What is the last bin? (b) What can you say about the mass of the lightest apple? Of the heaviest? (c) Mia says the heaviest apple is 145 g heavier than the lightest. Explain why that cannot be.

ANSWER

(a) 240 to 260 g.

(b) The lightest is in the first bin: at least 120 g and less than 140 g, so 120 to 139 g. The heaviest is in the last bin: 240 to 259 g.

(c) The biggest possible difference is 259 − 120 = 139 g, less than 145 g.

SOLUTION

(a) Seven bins of 20 g from 120 end at 120 + 7 × 20 = 260, so the last bin is 240 to 260.

(b) The first edge is the largest multiple of 20 at or below the lightest apple, so the lightest lies in the first bin: 120 ≤ mass < 140. In whole grams that is 120 to 139 g. Bins stop as soon as one holds the heaviest, so the heaviest lies in the last bin: 240 ≤ mass < 260, that is 240 to 259 g.

(c) The difference is largest when the heaviest is as heavy as it can be and the lightest as light as it can be: 259 − 120 = 139 g. A difference of 145 g would need a heavier apple (in a bin 260 to 280) or a lighter one (in a bin 100 to 120), and the histogram would then have more than 7 bins.

5. (a) Write 10 arm spans, in whole centimeters, so that a histogram with bins 10 cm wide starting at 130 (a value on an edge going in the bar to its right) has bars 2, 5, 0 and 3 tall, in that order, and at least three of your values sit on a bin edge.

(b) Explain why none of your values can be 150 or 170.

ANSWER

(a) One possible set: 130, 137, 140, 140, 144, 147, 149, 160, 163, 168. Any set works with 2 values from 130 to 139, 5 from 140 to 149, none from 150 to 159, 3 from 160 to 169, and at least three of them equal to 130, 140 or 160.

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(b) 150 would go in "150 to 160", which must be empty; 170 would go in "170 to 180", a fifth bin the histogram does not have.

SOLUTION

(a) The bins are 130 to 140, 140 to 150, 150 to 160 and 160 to 170. The set 130, 137, 140, 140, 144, 147, 149, 160, 163, 168 puts 130, 137 in the first bin (2), 140, 140, 144, 147, 149 in the second (5), nothing in the third (0) and 160, 163, 168 in the last (3). Four values, 130, 140, 140 and 160, sit on edges, and each goes in the bin it starts. Check: $2 + 5 + 0 + 3 = 10$.

(b) The edges that may be used are 130, 140 and 160. A value of 150 sits on an edge and goes in the bar to its right, "150 to 160", whose bar must have height 0. A value of 170 is not in "160 to 170", which leaves out its right edge, so it would need a bin "170 to 180" and a fifth bar.

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