Statistics Unit 1 worksheet: Box plots that show outliers (answer key)

ANSWERS AT A GLANCE

1. Five-number summary: 40, 48, 56, 62, 88. Fences: 27 and 83. Outlier: 88. The whiskers end at 40 and 63.
2. (a) 27, 33, 38, 45, 71; fences 15 and 63. (b) Only 71. (c) The whiskers end at 27 and 63.
3. (a) One list: 14, 17, 20, 23, 25, 26, 28, 30, 32, 38, 55. Any list works whose 1st, 3rd, 6th, 9th, 10th and 11th values are 14, 20, 26, 32, 38, 55, in order. (b) No. The upper fence is 50, so 48 would not be an outlier.
4. (a) Yes: the fences are 3 and 27, and 30 > 27. (b) No: the fences become −2 and 38, and 30 < 38. (c) The new value joined the upper half and pulled Q_3 from 18 up to 23, so the IQR grew from 6 to 10 and the upper fence moved from 27 out to 38.
5. 2. Example: 1, 2, 3, 4, 5, 6, 7, 8, 9, 100, 200, whose upper fence is 18.
6. She measured the fences from the median instead of from the quartiles, and she ended the upper whisker at a fence instead of at a data value. Correct fences 2.5 and 46.5; the outlier is 48; the whiskers end at 12 and 31.

WORKED SOLUTIONS

1. The weights, in kilograms, of the 11 dogs entered in the large-breed class at a dog show are listed in order: 40, 44, 48, 49, 53, 56, 58, 59, 62, 63, 88. Find the five-number summary, the fences, any outliers, and where each whisker of the modified box plot ends.

   ANSWER
   Five-number summary: 40, 48, 56, 62, 88. Fences: 27 and 83. Outlier: 88. The whiskers end at 40 and 63.

   SOLUTION
   With 11 values the median is the 6th, 56, left out of both halves. The lower half 40, 44, 48, 49, 53 gives $Q_1 = 48$, and the upper half 58, 59, 62, 63, 88 gives $Q_3 = 62$. The minimum is 40 and the maximum 88.

   IQR = 62 − 48 = 14 and 1.5 × 14 = 21, so the lower fence is 48 − 21 = 27 and the upper fence is 62 + 21 = 83.

   No weight is below 27, so the lower whisker runs to the minimum, 40. The weight 88 is above 83, so it is an outlier and gets a dot. The largest weight that is not an outlier is 63, so the upper whisker ends at 63, not at the fence.

2. A dog walker timed her 10 walks one week, in minutes, in order: 27, 29, 33, 34, 37, 39, 42, 45, 63, 71.

   (a) Find the five-number summary and the fences.

   (b) Which times are outliers?

   (c) Where does each whisker of the modified box plot end?

   ANSWER
   (a) 27, 33, 38, 45, 71; fences 15 and 63. (b) Only 71. (c) The whiskers end at 27 and 63.

   SOLUTION
   (a) With 10 values the median is the mean of the 5th and 6th, $\frac{37+39}{2} = 38$. The lower half 27, 29, 33, 34, 37 gives $Q_1 = 33$, and the upper half 39, 42, 45, 63, 71 gives $Q_3 = 45$. So IQR = 45 − 33 = 12, 1.5 × 12 = 18, and the fences are 33 − 18 = 15 and 45 + 18 = 63.

   (b) No time is below 15. The time 63 lies exactly on the upper fence, and a value on a fence is not an outlier: an outlier must be strictly beyond it. Only 71 is above 63, so 71 is the one outlier.

   (c) The lower whisker reaches the minimum, 27. The upper whisker runs to the largest time that is not an outlier, 63, and 71 is a dot.

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3. (a) Write a list of 11 whole numbers whose modified box plot has a box from 20 to 32 with its median at 26, whiskers ending at 14 and 38, and one dot, at 55. Show that your list gives this plot.

(b) Could the dot have been at 48 instead of 55, with the box and whiskers unchanged? Explain.

ANSWER

(a) One list: 14, 17, 20, 23, 25, 26, 28, 30, 32, 38, 55. Any list works whose 1st, 3rd, 6th, 9th, 10th and 11th values are 14, 20, 26, 32, 38, 55, in order. (b) No. The upper fence is 50, so 48 would not be an outlier.

SOLUTION

(a) With 11 values, Q_1 is the 3rd value, the median the 6th and Q_3 the 9th. So the 3rd, 6th and 9th values are 20, 26 and 32. The dot is the largest value, the 11th, 55. The whisker ends are values that are not outliers: the upper one, 38, is the 10th value, and the lower one, 14, is the 1st. The other places only need to keep the list in order, for example 14, 17, 20, 23, 25, 26, 28, 30, 32, 38, 55.

Check: IQR = 32 − 20 = 12, 1.5 × 12 = 18, so the fences are 20 − 18 = 2 and 32 + 18 = 50. Only 55 is beyond a fence, so it is the one dot; the whiskers reach 14 and 38.

(b) No. Moving the largest value to 48 leaves the quartiles, and so the fences 2 and 50, where they were. But 48 is not above 50, so it is not an outlier: the upper whisker would run out to 48 and there would be no dot.

4. Leah logged the pages she read on 7 days: 10, 12, 14, 15, 16, 18, 30.

(a) Is 30 an outlier?

(b) On an eighth day she read 28 pages. With all 8 days, is 30 an outlier?

(c) Explain how adding one value changed the answer.

ANSWER

(a) Yes: the fences are 3 and 27, and 30 > 27. (b) No: the fences become −2 and 38, and 30 < 38. (c) The new value joined the upper half and pulled Q_3 from 18 up to 23, so the IQR grew from 6 to 10 and the upper fence moved from 27 out to 38.

SOLUTION

(a) With 7 values the median is the 4th, 15. The lower half 10, 12, 14 gives $Q_1 = 12$ and the upper half 16, 18, 30 gives $Q_3 = 18$. IQR = 6, 1.5 × 6 = 9, and the fences are 12 − 9 = 3 and 18 + 9 = 27. Since 30 > 27, it is an outlier.

(b) In order: 10, 12, 14, 15, 16, 18, 28, 30. The lower half 10, 12, 14, 15 gives $Q_1 = \dfrac{12 + 14}{2} = 13$; the upper half 16, 18, 28, 30 gives $Q_3 = \dfrac{18 + 28}{2} = 23$. IQR = 10, 1.5 × 10 = 15, and the fences are 13 − 15 = −2 and 23 + 15 = 38. Now 30 is inside, so it is not an outlier.

(c) An outlier belongs to its data set. The value 28 landed in the upper half and moved Q_3 from 18 to 23, widening the box from 6 to 10 pages. A wider box moves the fences farther out, and the upper fence went from 27 to 38, past 30.

5. A data set has 11 values. What is the greatest number of them that can be outliers above the upper fence? Explain, and give a data set with that many.

ANSWER

2. Example: 1, 2, 3, 4, 5, 6, 7, 8, 9, 100, 200, whose upper fence is 18.

SOLUTION

With 11 values in order, Q_3 is the 9th value. The IQR is never negative, so the upper fence $Q_3 + 1.5\,\text{IQR}$ is at least Q_3. A high outlier is strictly above the fence, so strictly above Q_3, and only the 10th and 11th

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values can be above the 9th. So at most 2 values are high outliers.

Two is possible. In 1, 2, 3, 4, 5, 6, 7, 8, 9, 100, 200 the median is 6, $Q_1 = 3$ and $Q_3 = 9$, so IQR = 6, 1.5 × 6 = 9, and the upper fence is 9 + 9 = 18. Both 100 and 200 are above it.

6. Nine students did as many push-ups as they could in one minute: 12, 18, 20, 22, 25, 27, 29, 31, 48. Mei found the median 25 and the IQR 11 correctly, and 1.5 × 11 = 16.5. She then worked out the fences as 25 − 16.5 = 8.5 and 25 + 16.5 = 41.5, drew a dot at 48, drew the lower whisker to 12, and drew the upper whisker from the box out to 41.5.

Name Mei's two mistakes, and give the correct fences, outliers and whisker ends.

ANSWER

She measured the fences from the median instead of from the quartiles, and she ended the upper whisker at a fence instead of at a data value. Correct fences 2.5 and 46.5; the outlier is 48; the whiskers end at 12 and 31.

SOLUTION

The lower half 12, 18, 20, 22 gives $Q_1 = \dfrac{18 + 20}{2} = 19$ and the upper half 27, 29, 31, 48 gives $Q_3 = \dfrac{29 + 31}{2} = 30$; indeed IQR = 30 − 19 = 11.

Mistake 1: the fences are measured from the quartiles, not the median. They are 19 − 16.5 = 2.5 and 30 + 16.5 = 46.5.

Mistake 2: a whisker stops at a data value, the most extreme one that is not an outlier, never at a fence. No student did between 31 and 48 push-ups, and a whisker to 41.5 (or to 46.5) would say some did.

With the correct fences, no value is below 2.5 and only 48 is above 46.5, so 48 is the one outlier. The lower whisker ends at 12 and the upper at 31.

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