Statistics Unit 1 worksheet: Quartiles and the five number summary (answer key)

ANSWERS AT A GLANCE

1. (a) $Q_1 = 6$, median 11, $Q_3 = 17$ (b) $Q_1 = 15$, median 22.5, $Q_3 = 28$ (c) $Q_1 = 7.5$, median 11, $Q_3 = 16$ (d) $Q_1 = 43$, median 49.5, $Q_3 = 56$
2. (a) 95, 104, 113, 122, 140 beats per minute. (b) Range 45, IQR 18 beats per minute. (c) 3
3. (a) 5 (b) 10 (c) $\frac{1}{2}$ (d) Range 23 minutes, IQR 7 minutes.
4. (a) 13.25 hours (b) $Q_1 = 3$ hours, $Q_3 = 7$ hours (c) 9 of the 10 values; quartiles cut the sorted data into parts with equal counts, not the number line into equal lengths.
5. (a) 2, 3, 4, 5, 7 pairs. (b) 6 students; many students sit exactly on Q_1 or Q_3, so they are not counted as strictly between them.

WORKED SOLUTIONS

1. Find Q_1, the median and Q_3 of each list. Sort the values first, and leave the median out of both halves when the count is odd.

   (a) 14, 6, 11, 20, 9, 17, 3

   (b) 21, 15, 30, 12, 26, 18, 35, 24, 10, 28

   (c) 8, 15, 4, 12, 9, 20, 6, 11, 17, 7, 14, 10, 23

   (d) 40, 52, 47, 38, 61, 55, 44, 49, 58, 42, 50, 36, 63, 46, 53, 57

   ANSWER

   (a) $Q_1 = 6$, median 11, $Q_3 = 17$ (b) $Q_1 = 15$, median 22.5, $Q_3 = 28$ (c) $Q_1 = 7.5$, median 11, $Q_3 = 16$ (d) $Q_1 = 43$, median 49.5, $Q_3 = 56$

   SOLUTION

   (a) Sorted: 3, 6, 9, 11, 14, 17, 20. With 7 values the median is the 4th, 11, and it is left out. Lower half 3, 6, 9, so $Q_1 = 6$; upper half 14, 17, 20, so $Q_3 = 17$.

   (b) Sorted: 10, 12, 15, 18, 21, 24, 26, 28, 30, 35. With 10 values the median is $\frac{21 + 24}{2} = 22.5$. Lower half 10, 12, 15, 18, 21, so $Q_1 = 15$; upper half 24, 26, 28, 30, 35, so $Q_3 = 28$.

   (c) Sorted: 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 20, 23. With 13 values the median is the 7th, 11, and it is left out. Lower half 4, 6, 7, 8, 9, 10, so $Q_1 = \frac{7 + 8}{2} = 7.5$; upper half 12, 14, 15, 17, 20, 23, so $Q_3 = \frac{15 + 17}{2} = 16$.

   (d) Sorted: 36, 38, 40, 42, 44, 46, 47, 49, 50, 52, 53, 55, 57, 58, 61, 63. With 16 values the median is $\frac{49 + 50}{2} = 49.5$. Lower half 36, 38, 40, 42, 44, 46, 47, 49, so $Q_1 = \frac{42 + 44}{2} = 43$; upper half 50, 52, 53, 55, 57, 58, 61, 63, so $Q_3 = \frac{55 + 57}{2} = 56$.

2. Mason's gym class did jumping jacks for one minute, then each of the 14 students took a pulse. The heart rates, in beats per minute, were

   112, 98, 125, 104, 131, 117, 95, 120, 108, 140, 101, 114, 122, 110.

   (a) Find the five-number summary.

   (b) Find the range and the IQR.

   (c) How many of the 14 heart rates are below Q_1?

   ANSWER

   (a) 95, 104, 113, 122, 140 beats per minute. (b) Range 45, IQR 18 beats per minute. (c) 3

   SOLUTION

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(a) Sorted: 95, 98, 101, 104, 108, 110, 112, 114, 117, 120, 122, 125, 131, 140. With 14 values the median is $\frac{112+114}{2} = 113$. The lower half is the first seven values, 95, 98, 101, 104, 108, 110, 112, whose middle (fourth) value is $Q_1 = 104$. The upper half is 114, 117, 120, 122, 125, 131, 140, so $Q_3 = 122$. The five-number summary is 95, 104, 113, 122, 140 beats per minute.

(b) Range = 140 − 95 = 45 beats per minute; IQR = 122 − 104 = 18 beats per minute.

(c) Below 104 are 95, 98 and 101: 3 heart rates. (Q_1 is itself a data value here, so it sits on the cut and the bottom quarter holds 3, not 3.5.)

3. Layla timed how long each of the 20 students in her class waited in the lunch line on Monday. All 20 times are different. The five-number summary is 2, 5, 8, 12, 25 minutes.

(a) How many students waited less than 5 minutes?

(b) How many waited between 5 and 12 minutes?

(c) What fraction of the class waited more than 8 minutes?

(d) Find the range and the IQR.

ANSWER

(a) 5 (b) 10 (c) $\frac{1}{2}$ (d) Range 23 minutes, IQR 7 minutes.

SOLUTION

20 is a multiple of 4 and no two times are equal, so each quarter holds exactly $\frac{20}{4} = 5$ times, and no time sits on a cut: each quartile is halfway between two different times.

(a) The times below $Q_1 = 5$ are the lowest quarter: 5 students.

(b) From $Q_1 = 5$ to $Q_3 = 12$ are the two middle quarters: 10 students.

(c) Above the median 8 are the two top quarters, 10 of 20 students, which is $\frac{1}{2}$.

(d) Range = 25 − 2 = 23 minutes; IQR = 12 − 5 = 7 minutes.

4. The 10 members of Zoe's club did these numbers of hours of volunteer work this month:

5, 3, 8, 2, 47, 4, 6, 3, 7, 5.

Zoe says, "Q_1 is a quarter of the way from the minimum to the maximum, so I add a quarter of the range to the minimum."

(a) What number does Zoe's method give for Q_1?

(b) Find the real Q_1 and Q_3.

(c) How many of the 10 values are below Zoe's number? Explain what is wrong with her method.

ANSWER

(a) 13.25 hours (b) $Q_1 = 3$ hours, $Q_3 = 7$ hours (c) 9 of the 10 values; quartiles cut the sorted data into parts with equal counts, not the number line into equal lengths.

SOLUTION

(a) The range is 47 − 2 = 45, and a quarter of it is 11.25, so Zoe gets 2 + 11.25 = 13.25 hours.

(b) Sorted: 2, 3, 3, 4, 5, 5, 6, 7, 8, 47. With 10 values the lower half is 2, 3, 3, 4, 5, so $Q_1 = 3$, and the upper half is 5, 6, 7, 8, 47, so $Q_3 = 7$.

(c) Every value except 47 is below 13.25: 9 of the 10. A first quartile should have about a quarter of the values below it, not nine tenths. Zoe cut the number line from 2 to 47 into four equal lengths, but quartiles

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cut the sorted list into four parts with the same number of values. One far-off value like 47 stretches the lengths and pulls her number up, while the real Q_1 only looks at the lower half of the list.

5. Ryan asked 21 students how many pairs of shoes they own.

Pairs of shoes	**2**	**3**	**4**	**5**	**6**	**7**
Students	3	5	6	3	2	2

(a) Find the five-number summary.

(b) How many students own more than Q_1 pairs and fewer than Q_3 pairs? Explain why this is far from half of the 21 students.

ANSWER

(a) 2, 3, 4, 5, 7 pairs. (b) 6 students; many students sit exactly on Q_1 or Q_3, so they are not counted as strictly between them.

SOLUTION

(a) Running totals: 3, 8, 14, 17, 19, 21. So 2 pairs fills positions 1 to 3, 3 pairs positions 4 to 8, 4 pairs positions 9 to 14, 5 pairs positions 15 to 17, 6 pairs positions 18 and 19, and 7 pairs positions 20 and 21.

With $n = 21$ the median is position 11, which is 4 pairs, and it is left out of both halves. The lower half is positions 1 to 10; its median is halfway between positions 5 and 6, both 3, so $Q_1 = 3$. The upper half is positions 12 to 21; its median is halfway between positions 16 and 17, both 5, so $Q_3 = 5$. The five-number summary is 2, 3, 4, 5, 7 pairs.

(b) More than 3 and fewer than 5 means exactly 4 pairs: 6 students. Half of 21 would be about 10 or 11. From Q_1 to Q_3, counting the cuts, there are 5 + 6 + 3 = 14 students, but 8 of them own exactly 3 or exactly 5 pairs and sit on the cuts, so they are not strictly between. Quarters hold about equal counts only when few values repeat at the quartiles.

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