Statistics Unit 1 worksheet: Resistant and non resistant measures (answer key)

ANSWERS AT A GLANCE

1. (a) Mean 16 points, median 18 points. (b) Mean 18 points, median 18.5 points. (c) The mean moved more (2 points against 0.5 point).
2. (a) Mean 12 minutes, median 12 minutes, IQR 4 minutes. (b) Mean 16 minutes, median 12 minutes, IQR 5 minutes. (c) The median did not change, but the IQR moved from 4 to 5 minutes: resistant summaries move a little, not at all; only the mean jumped.
3. (a) Mean 30 minutes, MAD 2 minutes. (b) Mean 45 minutes, MAD 25 minutes. (c) IQR 5 minutes. (d) The IQR.
4. (a) Mean 7 books, median 6 books. (b) The median is 6 books if she reports 6 or fewer, and 6.5 books if she reports 7 or more. (c) Mean 11.3 books.
5. (a) True. (b) False. (c) False. Sample lists are in the solution; other lists can work.
6. (a) Mean 318 thousand dollars, median 248 thousand dollars. (b) One house. (c) The median, 248 thousand dollars, with the IQR, 20.5 thousand dollars, because the one price of 896 thousand dollars pulls the mean and the MAD far from the other eight.

WORKED SOLUTIONS

1. Chloe took nine spelling quizzes this term, each scored out of 20. Her scores were 17, 20, 15, 19, 0, 18, 16, 20, 19. The 0 is for a quiz she forgot to hand in, and her teacher agrees to drop it.

   (a) Find the mean and the median of all nine scores.

   (b) Find the mean and the median of the eight scores left after the 0 is dropped.

   (c) Which of the two summaries did dropping the 0 move more? Explain why in terms of what each summary uses.

   ANSWER

   (a) Mean 16 points, median 18 points. (b) Mean 18 points, median 18.5 points. (c) The mean moved more (2 points against 0.5 point).

   SOLUTION

   (a) Sorted: 0, 15, 16, 17, 18, 19, 19, 20, 20. The total is 144, so the mean is $\frac{144}{9} = 16$ points. Nine values, so the median is the fifth, 18 points.

   (b) Sorted: 15, 16, 17, 18, 19, 19, 20, 20. The total is still 144, so the mean is $\frac{144}{8} = 18$ points. The median is halfway between the fourth and fifth values: $\frac{18 + 19}{2} = 18.5$ points.

   (c) The mean rose by 2 points and the median by only 0.5 point. The mean shares the total among all the quizzes, and the 0 added nothing to the total while still counting as one of the nine shares, so it pulled every share down. The median only asks which score sits in the middle place; dropping the lowest score shifts the middle by half a place, to the next score up, however low the dropped score was.

2. Amara posts cooking videos online. Her last ten videos last, in minutes, 12, 9, 15, 8, 13, 11, 16, 10, 14, 12.

   (a) Find the mean, the median and the IQR of the ten lengths.

   (b) She then posts a livestream that lasts 56 minutes. Find the mean, the median and the IQR of the eleven lengths.

   (c) Amara says, "The median and the IQR are resistant, so adding a video cannot change them at all." What is right about her statement, and what is wrong?

   ANSWER

   (a) Mean 12 minutes, median 12 minutes, IQR 4 minutes. (b) Mean 16 minutes, median 12 minutes, IQR 5 minutes. (c) The median did not change, but the IQR moved from 4 to 5 minutes: resistant summaries move a little, not at all; only the mean jumped.

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SOLUTION

(a) Sorted: 8, 9, 10, 11, 12, 12, 13, 14, 15, 16. The total is 120, so the mean is $\frac{120}{10} = 12$ minutes. The median is halfway between the fifth and sixth values, $\frac{12 + 12}{2} = 12$ minutes. The halves are 8, 9, 10, 11, 12 and 12, 13, 14, 15, 16, so $Q_1 = 10$, $Q_3 = 14$ and the IQR is 14 − 10 = 4 minutes.

(b) Sorted: 8, 9, 10, 11, 12, 12, 13, 14, 15, 16, 56. The total is 176, so the mean is $\frac{176}{11} = 16$ minutes. Eleven values, so the median is the sixth, 12 minutes. Leaving it out, the halves are 8, 9, 10, 11, 12 and 13, 14, 15, 16, 56, so $Q_1 = 10$, $Q_3 = 15$ and the IQR is 15 − 10 = 5 minutes.

(c) She is right that the livestream's length does not matter to the median or the IQR: a livestream of 20 minutes or of 90 minutes would give the same median and IQR, because each only uses positions in the sorted list. She is wrong that they cannot change at all. A new value shifts the other values by one place, so Q_3 moved up to the next value, 15, and the IQR grew by 1 minute. (The median happened to stay because the two middle lengths were both 12.) The mean moved by 4 minutes and is now longer than nine of the eleven videos.

3. Diego practices sketching every day. From Monday to Friday he sketched for 25, 30, 35, 30, 30 minutes. On Saturday he spent 120 minutes at an art workshop.

(a) Find the mean and the MAD of the five weekday times.

(b) Find the mean and the MAD of all six days.

(c) Find the IQR of all six days.

(d) Diego wants one number for how much his sketching time usually changes from day to day. Should he report the MAD from (b) or the IQR from (c)? Explain.

ANSWER

(a) Mean 30 minutes, MAD 2 minutes. (b) Mean 45 minutes, MAD 25 minutes. (c) IQR 5 minutes. (d) The IQR.

SOLUTION

(a) The total is 150, so the mean is $\frac{150}{5} = 30$ minutes. The distances from 30 are 5, 0, 5, 0, 0, total 10, so the MAD is $\frac{10}{5} = 2$ minutes.

(b) The total is 150 + 120 = 270, so the mean is $\frac{270}{6} = 45$ minutes. The distances from 45 are 20, 15, 10, 15, 15, 75, total 150, so the MAD is $\frac{150}{6} = 25$ minutes.

(c) Sorted: 25, 30, 30, 30, 35, 120. The halves are 25, 30, 30 and 30, 35, 120, so $Q_1 = 30$ and $Q_3 = 35$. The IQR is 35 − 30 = 5 minutes.

(d) The IQR. A MAD of 25 minutes says the times typically sit 25 minutes from 45, but five of the six days lie between 25 and 35 minutes. The Saturday time raised the MAD in two ways: its own distance of 75 is one of the six distances averaged, and it moved the mean up to 45, away from the five weekdays. The IQR of 5 minutes uses only the middle of the sorted list, so it matches how much the ordinary days vary.

4. The nine members of a book club finished these numbers of books this summer: 4, 7, 5, 9, 6, 8, 5, 13, 6. A tenth member joins and reports her own number of books, a whole number.

(a) Find the mean and the median of the nine members.

(b) Whatever the tenth member reports, the median of the ten can be only one of two values. Find both, and say which reports give each.

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(c) Find the mean of the ten if the tenth member finished 50 books. Explain why her 50 moves the mean so much more than it moves the median.

ANSWER

(a) Mean 7 books, median 6 books. (b) The median is 6 books if she reports 6 or fewer, and 6.5 books if she reports 7 or more. (c) Mean 11.3 books.

SOLUTION

(a) Sorted: 4, 5, 5, 6, 6, 7, 8, 9, 13. The total is 63, so the mean is $\frac{63}{9} = 7$ books. The median is the fifth value, 6 books.

(b) With ten values the median is halfway between the fifth and sixth. If she reports 6 or fewer, her number goes at or before the two 6s, so the fifth and sixth values are 6 and 6 and the median is 6 books. If she reports 7 or more, the sorted list starts 4, 5, 5, 6, 6, 7, ..., so the fifth and sixth values are 6 and 7 and the median is $\frac{6+7}{2} = 6.5$ books. No other median is possible.

(c) The total is 63 + 50 = 113, so the mean is $\frac{113}{10} = 11.3$ books, more than eight of the first nine members read. The mean shares out the total, so all 50 books count. The median only asks which values sit in the middle places; the 50 goes last in the sorted list, just as a 7 would, so it gives the same median, 6.5 books.

5. Say whether each statement is true or false. For a true statement, give a list that shows it can happen. For a false statement, give a list that shows it is wrong.

(a) Raising the smallest value of a list can change Q_1.

(b) Doubling the largest value of a list doubles the range.

(c) If the mean and the median of a list are equal, the list has no outliers by the 1.5 IQR rule.

ANSWER

(a) True. (b) False. (c) False. Sample lists are in the solution; other lists can work.

SOLUTION

(a) True. Take 1, 2, 3, 4, 5, 6, 7, 8, with $Q_1 = \frac{2+3}{2} = 2.5$. Raise the 1 to 9: the list becomes 2, 3, 4, 5, 6, 7, 8, 9 and $Q_1 = \frac{3+4}{2} = 3.5$. Raising the smallest value far enough moves it to the other end, so the other values change places. (Lowering it never does, when the list has 6 or more values.)

(b) False. Take 2, 5, 10, with range 10 − 2 = 8. Doubling the 10 gives 2, 5, 20, with range 20 − 2 = 18, not 16. The range rises by the amount the largest value rises, here 10, not to twice the old range.

(c) False. Take 0, 19, 20, 21, 21, 22, 23, 42. The total is 168, so the mean is $\frac{168}{8} = 21$, and the median is $\frac{21+21}{2} = 21$. But $Q_1 = \frac{19+20}{2} = 19.5$ and $Q_3 = \frac{22+23}{2} = 22.5$, so the IQR is 3, the step is 1.5 × 3 = 4.5, and the fences are 15 and 27. Both 0 and 42 are outliers. Their pulls on the mean, one down and one up, cancel.

6. Nine houses on one street were sold last year. Their prices, in thousands of dollars, were 248, 236, 262, 230, 896, 251, 244, 255, 240.

(a) Find the mean and the median price.

(b) How many of the nine houses sold for more than the mean?

(c) A family thinking of moving to the street wants one number for what a house there usually costs, and one for how much the prices usually vary. Which center and which measure of spread should they use? Find the spread you choose, and explain your choice.

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ANSWER

(a) Mean 318 thousand dollars, median 248 thousand dollars. (b) One house. (c) The median, 248 thousand dollars, with the IQR, 20.5 thousand dollars, because the one price of 896 thousand dollars pulls the mean and the MAD far from the other eight.

SOLUTION

(a) Sorted: 230, 236, 240, 244, 248, 251, 255, 262, 896. The total is 2,862, so the mean is $\frac{2862}{9} = 318$ thousand dollars. Nine values, so the median is the fifth, 248 thousand dollars.

(b) Only the 896 is more than 318. The other eight prices are all 262 or less, so one house sold for more than the mean.

(c) Use the median and the IQR. Leaving out the median, the halves are 230, 236, 240, 244 and 251, 255, 262, 896, so $Q_1 = \frac{236 + 240}{2} = 238$ and $Q_3 = \frac{255 + 262}{2} = 258.5$. The IQR is 258.5 − 238 = 20.5 thousand dollars.

The mean of 318 thousand dollars is more than eight of the nine houses cost, so it does not describe a usual house; the one very expensive house raised it. The median and IQR use positions in the sorted list, so the 896 barely affects them, and they describe the eight ordinary sales well. The MAD is measured from the mean and would also be pushed up by the one far price.

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