Statistics Unit 9 worksheet: One sample test for a mean (answer key)

ANSWERS AT A GLANCE

1. μ = the mean weight of balm in all of the company's tubes on sale in the state; $H_0: \mu = 4.5$, $H_a: \mu < 4.5$. Random: a random sample of tubes. 10%: 15 is far less than 10% of many thousands of tubes. Normal/Large Sample: $n = 15$ is under 30, but the dot plot is roughly symmetric with no outliers. $\text{SE}_{\bar{x}} = 0.19/\sqrt{15} \approx 0.0491$, $t \approx -2.04$, df = 14, p-value ≈ 0.0304. Since 0.0304 is at most $\alpha = 0.05$, reject H_0: at the $\alpha = 0.05$ level there is convincing evidence that the mean weight of balm in the company's tubes is less than 4.5 grams.
2. (a) df = 6; 2.447 < 2.61 < 3.143, so 0.01 < p-value < 0.025; reject at $\alpha = 0.05$, fail to reject at $\alpha = 0.01$; technology ≈ 0.0201. (b) df = 13; 1.350 < 1.52 < 1.771, so 0.05 < p-value < 0.10; fail to reject at both levels; technology ≈ 0.0762. (c) df = 22; 2.074 < 2.30 < 2.508, so one tail is between 0.01 and 0.025 and 0.02 < p-value < 0.05; reject at $\alpha = 0.05$, fail to reject at $\alpha = 0.01$; technology ≈ 0.0313. (d) df = 6; 3.707 < 4.05 < 4.317, so one tail is between 0.0025 and 0.005 and 0.005 < p-value < 0.01; reject at both levels; technology ≈ 0.0067.
3. μ = the mean certification exam speed of all 400 of the school's graduates from the past five years; $H_0: \mu = 225$, $H_a: \mu > 225$. Random: a random sample of graduates. 10%: 10 ≤ 0.10(400) = 40. Normal/Large Sample: $n = 10$ is under 30; a dot plot of the speeds (220 to 244) is roughly symmetric with no outliers. $\bar{x} = 230.8$, $s_x \approx 7.5982$, $\text{SE}_{\bar{x}} \approx 2.4028$, $t \approx 2.41$, df = 9. Table: 2.262 < 2.41 < 2.821, so 0.01 < p-value < 0.025; technology ≈ 0.0195. Since 0.0195 ≤ 0.05, reject H_0: at the $\alpha = 0.05$ level there is convincing evidence that the school's graduates from the past five years average more than 225 words per minute on the certification exam.
4. (a) μ = the mean weight of filling in all of today's ravioli; $H_0: \mu = 12.0$, $H_a: \mu \neq 12.0$. (b) Random sample; 25 is far less than 10% of many thousands; $n = 25 < 30$ but the dot plot is roughly symmetric with no outliers. (c) $\text{SE}_{\bar{x}} = 0.80/\sqrt{25} = 0.16$, $t = 2.25$, df = 24, p-value ≈ 0.0339. (d) At $\alpha = 0.05$: 0.0339 ≤ 0.05, reject H_0, convincing evidence that today's mean filling is not 12.0 grams (the sample points to too much). At $\alpha = 0.01$: 0.0339 > 0.01, fail to reject. (e) 12.0 is outside the 95% interval, because the two-sided test at 0.05 rejects; it is inside the 99% interval, because the two-sided test at 0.01 does not reject. (f) $12.36 \pm 2.797(0.16) = (11.9125,\ 12.8075)$, which contains 12.0.
5. (a) $\text{SE}_{\bar{x}} = 4.4/\sqrt{16} = 1.1$, $t = (28.9 - 30)/1.1 = -1.00$, df = 15. (b) p-value $= P(T \geq -1.00) \approx 0.8334$. (c) H_a points up, so the p-value is the area to the right of the observed t; since t is negative, that area includes the whole right half of the symmetric curve plus the strip between −1.00 and 0, so it is more than 0.5. (d) 0.8334 > 0.05, fail to reject H_0: the data do not give convincing evidence that webs near porch lights have a mean diameter above 30 centimeters. (e) The alternative is fixed before the data are seen; choosing it after seeing which way the data fell makes the p-value look smaller without any new evidence. A question about smaller webs would need new data.

WORKED SOLUTIONS

1. A made-up cosmetics company labels each tube of its lip balm as holding 4.5 grams. A consumer group suspects that the tubes hold less than that on average. Leah buys a random sample of 15 tubes from stores across the state, which stock many thousands of the tubes, and weighs the balm in each one. The sample gives $\bar{x} = 4.40$ grams and $s_x = 0.19$ grams, and a dot plot of the 15 weights is roughly symmetric with no outliers.

   Carry out the test at $\alpha = 0.05$: define the parameter, state the hypotheses, check the conditions, find $\text{SE}_{\bar{x}}$ (four decimal places), t (two decimal places, from the unrounded $\text{SE}_{\bar{x}}$) and the degrees of freedom, find the p-value with technology (four decimal places), and give the decision and a conclusion in context.

   ANSWER

   μ = the mean weight of balm in all of the company's tubes on sale in the state; $H_0: \mu = 4.5$, $H_a: \mu < 4.5$. Random: a random sample of tubes. 10%: 15 is far less than 10% of many thousands of tubes. Normal/Large Sample: $n = 15$ is under 30, but the dot plot is roughly symmetric with no outliers. $\text{SE}_{\bar{x}} = 0.19/\sqrt{15} \approx 0.0491$, $t \approx -2.04$, df = 14, p-value ≈ 0.0304. Since 0.0304 is at most $\alpha = 0.05$, reject H_0: at the $\alpha = 0.05$ level there is convincing evidence that the mean weight of balm in the company's tubes is less than 4.5 grams.

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SOLUTION

Parameter: μ is the mean weight of balm, in grams, in all of the company's tubes on sale in the state. The consumer group's suspicion is the alternative:

$$H_0\colon \mu = 4.5 \qquad H_a\colon \mu < 4.5$$

Conditions. Random: the tubes were a random sample. 10%: 15 tubes is far less than a tenth of the many thousands on sale, so the weights are close enough to independent. Normal/Large Sample: $n = 15$ is under 30, so the shape matters; the dot plot is roughly symmetric with no outliers, so a t procedure is reasonable.

$$\mathrm{SE}_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{0.19}{\sqrt{15}} \approx 0.0491 \qquad t = \frac{4.40 - 4.5}{\mathrm{SE}_{\bar{x}}} \approx -2.04, \qquad \mathrm{df} = 15 - 1 = 14$$

The alternative points down, so the p-value is the lower tail of the t curve with 14 degrees of freedom: tcdf(-10^99, t, 14), with the unrounded t, gives about 0.0304.

Decision: the p-value 0.0304 is at most 0.05, so reject H_0. If the mean fill were 4.5 grams, a random sample of 15 tubes would average 4.40 grams or less with probability about 0.0304. At the $\alpha = 0.05$ level there is convincing evidence that the mean weight of balm in the company's tubes is less than 4.5 grams. Rejecting H_0 says the shortfall is hard to explain by chance alone; it does not say how large the shortfall is or whether it matters to a buyer.

2. Each row below is a one-sample t test for a mean, with its alternative, its sample size and the observed t.

Test	H_a	n	t
(a)	$\mu > \mu_0$	7	2.61
(b)	$\mu < \mu_0$	14	−1.52
(c)	$\mu \neq \mu_0$	23	2.30
(d)	$\mu \neq \mu_0$	7	−4.05

These are three rows of a t table. Each entry is the value of t with the given upper-tail area.

df	0.10	0.05	0.025	0.01	0.005	0.0025
6	1.440	1.943	2.447	3.143	3.707	4.317
13	1.350	1.771	2.160	2.650	3.012	3.372
22	1.321	1.717	2.074	2.508	2.819	3.119

For each test, give the degrees of freedom, use the table to bracket the p-value between two values, and decide at $\alpha = 0.05$ and at $\alpha = 0.01$. Then find each p-value with technology (four decimal places).

ANSWER

(a) df = 6; 2.447 < 2.61 < 3.143, so 0.01 < p-value < 0.025; reject at $\alpha = 0.05$, fail to reject at $\alpha = 0.01$; technology ≈ 0.0201. (b) df = 13; 1.350 < 1.52 < 1.771, so 0.05 < p-value < 0.10; fail to reject at both levels; technology ≈ 0.0762. (c) df = 22; 2.074 < 2.30 < 2.508, so one tail is between 0.01 and 0.025 and 0.02 < p-value < 0.05; reject at $\alpha = 0.05$, fail to reject at $\alpha = 0.01$; technology ≈ 0.0313. (d) df = 6; 3.707 < 4.05 < 4.317, so one tail is between 0.0025 and 0.005 and 0.005 < p-value < 0.01; reject at both levels; technology ≈ 0.0067.

SOLUTION

The degrees of freedom are $n - 1$. The table lists upper tails, and the t curve is symmetric, so a negative t is looked up by its size. A larger t sits further out, so its tail is smaller.

(a) df = 6. The alternative points up and $t = 2.61$ lies between 2.447 (tail 0.025) and 3.143 (tail 0.01), so 0.01 < p-value < 0.025. Every value in that range is at most 0.05 and above 0.01: reject H_0 at $\alpha = 0.05$, fail

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to reject at $\alpha = 0.01$. Technology: tcdf(2.61, 10^99, 6) ≈ 0.0201.

(b) df = 13. The alternative points down and $t = -1.52$ is on that side; its size lies between 1.350 (tail 0.10) and 1.771 (tail 0.05), so 0.05 < p-value < 0.10. Fail to reject at both levels. Technology: tcdf(-10^99, -1.52, 13) ≈ 0.0762.

(c) df = 22. Both tails count. 2.30 lies between 2.074 and 2.508, so one tail is between 0.01 and 0.025, and two tails between 0.02 and 0.05. Reject at $\alpha = 0.05$, fail to reject at $\alpha = 0.01$. Technology: 2 × tcdf(2.30, 10^99, 22) ≈ 0.0313.

(d) df = 6. Both tails count. $|t| = 4.05$ lies between 3.707 and 4.317, so one tail is between 0.0025 and 0.005, and two tails between 0.005 and 0.01. Reject at both levels. Technology: 2 × tcdf(-10^99, -4.05, 6) ≈ 0.0067.

In every case the bracket alone decides at both levels, because neither 0.05 nor 0.01 falls inside it.

3. A state's court reporter certificate requires a typing speed of 225 words per minute on a stenotype machine. A made-up stenography school's brochure claims that its graduates average more than that on the certification exam. Naomi, a journalist, picks 10 of the school's 400 graduates from the past five years at random and looks up each one's exam speed, in words per minute:

222, 229, 237, 220, 233, 244, 227, 231, 226, 239

Test the brochure's claim at $\alpha = 0.05$. Define the parameter, state the hypotheses, check the conditions (judge the shape from the data), compute $\bar{x}$, s_x (four decimal places), $\mathrm{SE}_{\bar{x}}$ (four decimal places), t (two decimal places) and the degrees of freedom. Bracket the p-value from this row of the t table, then find it with technology (four decimal places), and give the decision and a conclusion in context.

Upper-tail area	0.05	0.025	0.01	0.005
t for df = 9	1.833	2.262	2.821	3.250

ANSWER

μ = the mean certification exam speed of all 400 of the school's graduates from the past five years; $H_0\colon \mu = 225$, $H_a\colon \mu > 225$. Random: a random sample of graduates. 10%: 10 ≤ 0.10(400) = 40. Normal/Large Sample: $n = 10$ is under 30; a dot plot of the speeds (220 to 244) is roughly symmetric with no outliers. $\bar{x} = 230.8$, $s_x \approx 7.5982$, $\mathrm{SE}_{\bar{x}} \approx 2.4028$, $t \approx 2.41$, df = 9. Table: 2.262 < 2.41 < 2.821, so 0.01 < p-value < 0.025; technology ≈ 0.0195. Since 0.0195 ≤ 0.05, reject H_0: at the $\alpha = 0.05$ level there is convincing evidence that the school's graduates from the past five years average more than 225 words per minute on the certification exam.

SOLUTION

Parameter: μ is the mean certification exam speed, in words per minute, of all 400 of the school's graduates from the past five years. The brochure's claim is the alternative:

$$H_0\colon \mu = 225 \qquad H_a\colon \mu > 225$$

Conditions. Random: the 10 graduates were picked at random. 10%: 10 is at most 0.10(400) = 40, so the speeds are close enough to independent. Normal/Large Sample: $n = 10$ is under 30, so the data must look roughly normal. On a dot plot the speeds run from 220 to 244, spread fairly evenly on both sides of the middle with no value far from the rest: no strong skew and no outliers.

The speeds add to 2,308, so $\bar{x} = 230.8$. The squared deviations add to 519.6, so $s_x = \sqrt{519.6/9} \approx 7.5982$.

$$\mathrm{SE}_{\bar{x}} = \frac{7.5982}{\sqrt{10}} \approx 2.4028 \qquad t = \frac{230.8 - 225}{2.4028} \approx 2.41, \qquad \mathrm{df} = 9$$

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The alternative points up, so the p-value is the upper tail. In the row for df = 9, 2.262 < 2.41 < 2.821, so 0.01 < p-value < 0.025. Technology, tcdf(2.414, 10^99, 9), gives about 0.0195.

Decision: 0.0195 ≤ 0.05 (the whole bracket is below 0.05 too), so reject H_0. If the graduates' mean speed were 225 words per minute, a random sample of 10 would average 230.8 or more only about 2% of the time. At the $\alpha = 0.05$ level there is convincing evidence that the school's graduates from the past five years average more than 225 words per minute on the certification exam. The test is about the mean: it does not say that every graduate passed the 225 mark (one of the ten did not).

4. A made-up pasta factory makes many thousands of ravioli a day and aims for a mean of 12.0 grams of filling per ravioli: too little disappoints customers and too much costs money. Carlos, the quality manager, asks whether today's mean is off target in either direction. He weighs the filling in a random sample of 25 of today's ravioli and gets $\bar{x} = 12.36$ grams and $s_x = 0.80$ grams; a dot plot of the weights is roughly symmetric with no outliers.

(a) Define the parameter and state the hypotheses.

(b) Check the conditions.

(c) Find $\text{SE}_{\bar{x}}$, t (two decimal places), the degrees of freedom and the p-value (technology, four decimal places).

(d) Decide at $\alpha = 0.05$ and at $\alpha = 0.01$.

(e) Without building any interval, say whether 12.0 lies inside the 95% t interval for μ from these data, and whether it lies inside the 99% t interval. Explain.

(f) Build the 99% t interval ($t^* = 2.797$ for df = 24) and confirm your answer to (e).

ANSWER

(a) μ = the mean weight of filling in all of today's ravioli; $H_0\colon \mu = 12.0$, $H_a\colon \mu \neq 12.0$. (b) Random sample; 25 is far less than 10% of many thousands; $n = 25 < 30$ but the dot plot is roughly symmetric with no outliers. (c) $\text{SE}_{\bar{x}} = 0.80/\sqrt{25} = 0.16$, $t = 2.25$, df = 24, p-value ≈ 0.0339. (d) At $\alpha = 0.05$: 0.0339 ≤ 0.05, reject H_0, convincing evidence that today's mean filling is not 12.0 grams (the sample points to too much). At $\alpha = 0.01$: 0.0339 > 0.01, fail to reject. (e) 12.0 is outside the 95% interval, because the two-sided test at 0.05 rejects; it is inside the 99% interval, because the two-sided test at 0.01 does not reject. (f) $12.36 \pm 2.797(0.16) = (11.9125,\ 12.8075)$, which contains 12.0.

SOLUTION

(a) Let μ be the mean weight of filling, in grams, in all the ravioli made today. Carlos asks about a change in either direction, so

$$H_0\colon \mu = 12.0 \qquad H_a\colon \mu \neq 12.0$$

(b) Random: a random sample of today's ravioli. 10%: 25 is far below a tenth of many thousands. Normal/Large Sample: $n = 25$ is under 30, but the dot plot shows no strong skew and no outliers.

(c)

$$\text{SE}_{\bar{x}} = \frac{0.80}{\sqrt{25}} = 0.16 \qquad t = \frac{12.36 - 12.0}{0.16} = 2.25, \qquad \text{df} = 24$$

Both tails count: 2 × tcdf(2.25, 10^99, 24) ≈ 0.0339.

(d) At $\alpha = 0.05$, 0.0339 ≤ 0.05, so reject H_0: there is convincing evidence that today's mean filling weight is not 12.0 grams, and the sample points to too much filling. At $\alpha = 0.01$, 0.0339 > 0.01, so fail to reject H_0: at that stricter level the data do not give convincing evidence that the mean is off target.

(e) A two-sided test at level α rejects μ_0 exactly when μ_0 lies outside the $(1 - \alpha)$ t interval, because both use the same $\text{SE}_{\bar{x}}$ and the same t curve. The test at 0.05 rejects, so 12.0 is outside the 95% interval. The

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test at 0.01 does not reject, so 12.0 is inside the 99% interval.

(f)

$$12.36 \pm 2.797(0.16) = 12.36 \pm 0.4475 = (11.9125,\ 12.8075)$$

This contains 12.0, as (e) said. (For comparison, the 95% interval, with $t^* = 2.064$, is (12.0298, 12.6902), which leaves 12.0 out.)

5. A field guide gives 30 centimeters as the mean diameter of the webs of a made-up orb-weaver spider. Mia suspects that webs spun near porch lights, where insects gather, are larger on average. Before measuring anything she chose $H_a:\ \mu > 30$ and $\alpha = 0.05$, where μ is the mean diameter of all webs of this species near porch lights in her town. Her town's nature club has mapped about 300 such webs, and she measures a random sample of 16 of them: $\bar{x} = 28.9$ centimeters and $s_x = 4.4$ centimeters, with a roughly symmetric dot plot and no outliers.

(a) Find t and the degrees of freedom.

(b) Find the p-value with technology (four decimal places).

(c) Explain, using the t curve, why this p-value is greater than 0.5.

(d) Give the decision and a conclusion in context.

(e) A friend says: "The webs came out smaller than 30, so report a test of $H_a:\ \mu < 30$ instead." Explain why Mia should not.

ANSWER

(a) $\text{SE}_{\bar{x}} = 4.4/\sqrt{16} = 1.1$, $t = (28.9 - 30)/1.1 = -1.00$, df = 15. (b) p-value $= P(T \geq -1.00) \approx 0.8334$. (c) H_a points up, so the p-value is the area to the right of the observed t; since t is negative, that area includes the whole right half of the symmetric curve plus the strip between −1.00 and 0, so it is more than 0.5. (d) 0.8334 > 0.05, fail to reject H_0: the data do not give convincing evidence that webs near porch lights have a mean diameter above 30 centimeters. (e) The alternative is fixed before the data are seen; choosing it after seeing which way the data fell makes the p-value look smaller without any new evidence. A question about smaller webs would need new data.

SOLUTION

(a)

$$\text{SE}_{\bar{x}} = \frac{4.4}{\sqrt{16}} = 1.1 \qquad t = \frac{28.9 - 30}{1.1} = -1.00, \qquad \text{df} = 15$$

(b) The alternative is $\mu > 30$, so the p-value is the area at or above the observed t: tcdf(-1, 10^99, 15) ≈ 0.8334.

(c) The t curve is symmetric about 0, so the area to the right of 0 is 0.5. The observed $t = -1.00$ is to the left of 0, so the area to its right is that 0.5 plus the area between −1.00 and 0 (about 0.33). A sample mean on the opposite side of μ_0 from the alternative can never be evidence for that alternative.

(d) 0.8334 > 0.05, so Mia fails to reject H_0. At the $\alpha = 0.05$ level the data do not give convincing evidence that the mean diameter of webs near porch lights in her town is above 30 centimeters. This is not evidence that the mean is exactly 30 centimeters.

(e) The alternative and α belong to the question Mia asked before collecting data. Switching to $H_a:\ \mu < 30$ because the sample fell below 30 would give a lower-tail p-value of about 0.17 for the same data, but that is choosing the test to suit the data, which halves (or here, shrinks) the p-value without new evidence. If she now wonders whether porch-light webs are smaller, she should state that question and test it with a new random sample.

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