Statistics Unit 9 worksheet: One sample test for a mean

Name _____ Date _____ Period _____

1. A made-up cosmetics company labels each tube of its lip balm as holding 4.5 grams. A consumer group suspects that the tubes hold less than that on average. Leah buys a random sample of 15 tubes from stores across the state, which stock many thousands of the tubes, and weighs the balm in each one. The sample gives $\bar{x} = 4.40$ grams and $s_x = 0.19$ grams, and a dot plot of the 15 weights is roughly symmetric with no outliers.

Carry out the test at $\alpha = 0.05$: define the parameter, state the hypotheses, check the conditions, find $\text{SE}_{\bar{x}}$ (four decimal places), t (two decimal places, from the unrounded $\text{SE}_{\bar{x}}$) and the degrees of freedom, find the p-value with technology (four decimal places), and give the decision and a conclusion in context.

2. Each row below is a one-sample t test for a mean, with its alternative, its sample size and the observed t.

Test	H_a	n	t
(a)	$\mu > \mu_0$	7	2.61
(b)	$\mu < \mu_0$	14	−1.52
(c)	$\mu \neq \mu_0$	23	2.30
(d)	$\mu \neq \mu_0$	7	−4.05

These are three rows of a t table. Each entry is the value of t with the given upper-tail area.

df	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.0025**
6	1.440	1.943	2.447	3.143	3.707	4.317
13	1.350	1.771	2.160	2.650	3.012	3.372
22	1.321	1.717	2.074	2.508	2.819	3.119

For each test, give the degrees of freedom, use the table to bracket the p-value between two values, and decide at $\alpha = 0.05$ and at $\alpha = 0.01$. Then find each p-value with technology (four decimal places).

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3. A state's court reporter certificate requires a typing speed of 225 words per minute on a stenotype machine. A made-up stenography school's brochure claims that its graduates average more than that on the certification exam. Naomi, a journalist, picks 10 of the school's 400 graduates from the past five years at random and looks up each one's exam speed, in words per minute:

222, 229, 237, 220, 233, 244, 227, 231, 226, 239

Test the brochure's claim at $\alpha = 0.05$. Define the parameter, state the hypotheses, check the conditions (judge the shape from the data), compute $\bar{x}$, s_x (four decimal places), $\mathrm{SE}_{\bar{x}}$ (four decimal places), t (two decimal places) and the degrees of freedom. Bracket the p-value from this row of the t table, then find it with technology (four decimal places), and give the decision and a conclusion in context.

Upper-tail area	**0.05**	**0.025**	**0.01**	**0.005**
t for df = 9	1.833	2.262	2.821	3.250

4. A made-up pasta factory makes many thousands of ravioli a day and aims for a mean of 12.0 grams of filling per ravioli: too little disappoints customers and too much costs money. Carlos, the quality manager, asks whether today's mean is off target in either direction. He weighs the filling in a random sample of 25 of today's ravioli and gets $\bar{x} = 12.36$ grams and $s_x = 0.80$ grams; a dot plot of the weights is roughly symmetric with no outliers.

(a) Define the parameter and state the hypotheses.

(b) Check the conditions.

(c) Find $\mathrm{SE}_{\bar{x}}$, t (two decimal places), the degrees of freedom and the p-value (technology, four decimal places).

(d) Decide at $\alpha = 0.05$ and at $\alpha = 0.01$.

(e) Without building any interval, say whether 12.0 lies inside the 95% t interval for μ from these data, and whether it lies inside the 99% t interval. Explain.

(f) Build the 99% t interval ($t^* = 2.797$ for df = 24) and confirm your answer to (e).

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5. A field guide gives 30 centimeters as the mean diameter of the webs of a made-up orb-weaver spider. Mia suspects that webs spun near porch lights, where insects gather, are larger on average. Before measuring anything she chose $H_a:\ \mu > 30$ and $\alpha = 0.05$, where μ is the mean diameter of all webs of this species near porch lights in her town. Her town's nature club has mapped about 300 such webs, and she measures a random sample of 16 of them: $\bar{x} = 28.9$ centimeters and $s_x = 4.4$ centimeters, with a roughly symmetric dot plot and no outliers.

(a) Find t and the degrees of freedom.

(b) Find the p-value with technology (four decimal places).

(c) Explain, using the t curve, why this p-value is greater than 0.5.

(d) Give the decision and a conclusion in context.

(e) A friend says: "The webs came out smaller than 30, so report a test of $H_a:\ \mu < 30$ instead." Explain why Mia should not.

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